PHILOSOPHICAL

TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCVII.

PART I.

LONDON,

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ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to

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be, the importance and singularity of the subjects, or the advantageous manner of treating them, without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they receive them, are to be considered in no other light than as a matter of civility, in return for the respect shewn to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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Meteorological Journal kept at the Apartments of the Royal Society, by Order of the President and Council.

The PRESIDENT and COUNCIL of the ROYAL SOCIETY adjudged the Medal on Sir GODFREY COPLEY's Donation for the year 1806, to THOMAS ANDREW KNIGHT, Esq. F. R. S. for his various papers on Vegetation printed in the Philosophical Transactions.

PHILOSOPHICAL TRANSACTIONS.

I. *The Bakerian Lecture, on some chemical Agencies of Electricity.* By Humphry Davy, *Esq. F. R. S. M. R. I. A.*

Read November 20, 1806.

1. *Introduction.*

THE chemical effects produced by electricity have been for some time objects of philosophical attention; but the novelty of the phenomena, their want of analogy to known facts, and the apparent discordance of some of the results, have involved the enquiry in much obscurity.

An attempt to elucidate the subject will not, I hope, be considered by the Society as unfitted to the design of the Bakerian Lecture. I shall have to detail some minute (and I fear tedious) experiments; but they were absolutely essential to the investigation. I shall likewise, however, be able to offer some illustrations of appearances which hitherto have not been fully explained, and to point out some new properties of one of the most powerful and general of material agents.

II. *On the Changes produced by Electricity in Water.*

The appearance of acid and alkaline matter in water acted on by a current of electricity, at the opposite electrified metallic surfaces was observed in the first chemical experiments made with the column of VOLTA.*

Mr. CRUICKSHANK† supposed that the acid was the nitrous acid and the alkali ammonia. M. DESORMES‡ soon after attempted to show by experiments, that muriatic acid and ammonia were the products, and M. BRUGNATELLI§ asserted the formation of a new and peculiar substance, which he has thought proper to call the electric acid. The experiments said to be made in Italy, and in this country, on the production of muriate of soda are recent,‖ and the discussions with regard to them still alive. As early as 1800, I had found that when separate portions of distilled water, filling two glass tubes connected by moist bladders, or any moist animal or vegetable substances, were submitted to the electrical action of the pile of VOLTA by means of gold wires, a nitro-muriatic solution of gold appeared in the tube containing the positive wire, or the wire transmitting the electricity, and a solution of soda in the opposite tube;¶ but I soon ascertained that the muriatic acid owed its appearance to the animal or vegetable matters employed; for when the same fibres of cotton were made use of in successive experiments, and washed after every process in a weak solution of nitric acid, the water in the apparatus

* NICHOLSON'S Journal, 4to. Vol. IV. p. 183. † Ibid. Vol. IV. p. 261.

‡ *Annales de Chimie*, Tom. XXXVII. p. 233. § Phil. Mag. Vol. IX. p. 181.

‖ By M. PACCHIONI, and by Mr. PEELE. Phil. Mag. Vol. XXI. p. 279.

¶ I shewed the results of the experiment to Dr. BEDDOES at this time; and mentioned the circumstance to Sir JAMES HALL, Mr. CLAYFIELD, and other friends in 1801.

containing them, though acted on for a great length of time with a very strong power, at last produced no effect upon solution of nitrate of silver.

In cases when I had procured much soda, the glass at its point of contact with the wire seemed considerably corroded; and I was confirmed in my idea of referring the production of the alkali principally to this source, by finding that no fixed saline matter could be obtained, by electrifying distilled water in a single agate cup from two points of platina connected with the VOLTAIC battery. Similar conclusions with regard to the appearance of the muriatic acid had been formed by the GALVANIC Society of Paris, by Dr. WOLLASTON, who hit upon the happy expedient of connecting the tubes together by well washed asbestus; and by M. M. BIOT and THENARD.*

Mr. SYLVESTER, however, in a Paper published in Mr. NICHOLSON's Journal for last August, states, that though no fixed alkali or muriatic acid appears when a single vessel is employed; yet that they are both formed when two vessels are used. And to do away all objections with regard to vegetable substances or glass, he conducted his process in a vessel made of baked tobacco-pipe clay inserted in a crucible of platina. I have no doubt of the correctness of his results: but the conclusion appears objectionable. He conceives that he obtained fixed alkali, because the fluid after being heated and evaporated left a matter that tinged turmeric brown, which would have happened had it been lime, a substance that exists in considerable quantities in all pipe-clay; and even allowing the presence of fixed alkali, the materials employed for the manufacture of tobacco-pipes are not at all such as to exclude the combinations of this substance.

* No. XL. *Du Moniteur*, 1806.

I resumed the enquiry; I procured small cylindrical cups of agate, of the capacity of about $\frac{1}{4}$ of a cubic inch each. They were boiled for some hours in distilled water, and a piece of very white and transparent amianthus that had been treated in the same way was made to connect them together; they were filled with distilled water, and exposed by means of two platina wires to a current of electricity, from 150 pairs of plates of copper and zinc 4 inches square, made active by means of solution of alum. After 48 hours the process was examined: paper tinged with litmus plunged into the tube containing the transmitting or positive wire was immediately strongly reddened. Paper coloured by turmeric introduced into the other tube had its colour much deepened; the acid matter gave a very slight degree of turbidness to solution of nitrate of silver. The fluid that affected turmeric retained this property after being strongly boiled; and it appeared more vivid as the quantity became reduced by evaporation; carbonate of ammonia was mixed with it, and the whole dried and exposed to a strong heat: a minute quantity of white matter remained, which, as far as my examination could go, had the properties of carbonate of soda. I compared it with similar minute portions of the pure carbonates of potash and soda. It was not so deliquescent as the former of these bodies, and it formed a salt with nitric acid, which like nitrate of soda soon attracted moisture from a damp atmosphere, and became fluid.

This result was unexpected, but it was far from convincing me that the substances which I had obtained were *generated*. In a similar process, with glass tubes, carried on exactly under the same circumstances, and for the same time, I obtained a quantity of alkali which must have been more than twenty

times greater, but no traces of muriatic acid. There was much probability that the agate might contain some minute portion of saline matter, not easily detected by chemical analysis, either in combination, or intimate adhesion in its pores. To determine this, I repeated the experiment a second, a third, and a fourth time. In the second experiment turbidness was still produced by solution of nitrate of silver in the tube containing the acid, but it was less distinct; in the third process it was barely perceptible: and in the fourth the two fluids remained perfectly clear after the mixture. The quantity of alkaline matter diminished in every operation; and in the last process, though the battery had been kept in great activity for three days, the fluid possessed in a very slight degree only the power of acting on paper tinged with turmeric; but its alkaline property was very sensible to litmus paper slightly reddened, which is a much more delicate test: and after evaporation and the process by carbonate of ammonia, a barely perceptible quantity of fixed alkali was still left. The acid matter in the other tube was abundant; its taste was sour; it smelt like water over which large quantities of nitrous gas have been long kept; it did not affect solution of muriate of barytes; and a drop of it placed upon a polished plate of silver left after evaporation a black stain, precisely similar to that produced by extremely diluted nitrous acid.

After these results, I could no longer doubt that some saline matter existing in the agate tubes had been the source of the acid matter capable of precipitating nitrate of silver, and of much of the alkali. Four additional repetitions of the process, however, convinced me that there was likewise some other cause for the presence of this last substance; for it

continued to appear to the last, in quantities sufficiently distinguishable, and apparently equal in every case. I had used every precaution; I had included the tubes in glass vessels out of the reach of the circulating air; all the acting materials had been repeatedly washed with distilled water; and no part of them in contact with the fluid had been touched by the fingers.

The only substance which I could now conceive capable of furnishing the fixed alkali was the *water itself*. This water appeared pure by the tests of nitrate of silver and muriate of barytes; but potash and soda, as is well known, rise in small quantities in rapid distillations; and the New River water, which I made use of, contains animal and vegetable impurities, which it was easy to conceive might furnish neutral salts capable of being carried over in vivid ebullition.

To make the experiment in as refined a form as possible, I procured two hollow cones of pure gold containing about 25 grains of water each, they were filled with distilled water, connected together by a moistened piece of amianthus which had been used in the former experiments, and exposed to the action of a VOLTAIC battery of 100 pairs of plates of copper and zinc of 6 inches square, in which the fluid was a solution of alum and diluted sulphuric acid. In ten minutes the water in the negative tube had gained the power of giving a slight blue tint to litmus paper: and the water in the positive tube rendered it red. The process was continued for 14 hours; the acid increased in quantity during the whole time, and the water became at last very sour to the taste. The alkaline properties of the fluid in the other tube, on the contrary, remained stationary, and at the end of the time, it did not act upon litmus or turmeric paper more than in the first trial: the effect was less vivid after it had been strongly heated for a

minute; but evaporation and the usual process proved that *some* fixed alkali was present. The acid, as far as its properties were examined, agreed with pure nitrous acid having an excess of nitrous gas.

I *repeated* the experiment, and carried on the process for three days; at the end of which time the water in the tube was decomposed and evaporated to more than one half of its original quantity; the acid was strong, but the alkali in as minute a portion as in the last experiment. It acted indeed rather more vividly on the tests, on account of the greater diminution of the fluid, but presented the same results after being heated.

It was now impossible to doubt that the water contained some substance in very minute quantities, capable of causing the appearance of fixed alkali, but which was soon exhausted; and the question that immediately presented itself was, Is this substance saline matter carried over in distillation? or is it nitrogen gas which exists in minute portions in all water that has been exposed to air, and which, if an element of the fixed alkali, would under the circumstance of the experiment have been soon exhausted, whilst its absorption from the atmosphere would be impeded by the saturation of the water with hydrogen?

I was much more inclined to the former than to the latter supposition. I evaporated a quart of the distilled water that I had used, very slowly at a heat below 140° FAHRENHEIT, in a silver still; a solid matter remained, equal to $\frac{7}{10}$ of a grain; this matter had a saline but metallic taste, and was deliquescent when exposed to air: I could not obtain from it regular crystals; it did not affect turmeric or litmus, but a part

of it, after being heated red, in a silver crucible, exhibited strong alkaline properties. It was not possible to make a minute analysis of so small a quantity, but it appeared to me to be principally a mixture of nitrate of soda and nitrate of lead; and the metallic substance, it is most likely, was furnished by the condensing tube of the common still.

The existence of saline matter in the distilled water being thus distinct, it was easy to determine its operation in the experiment. I filled the two gold cones with water in the usual manner; that negatively electrified, soon attained the maximum of its effect upon turmeric paper. I then introduced into it a very minute portion of the substance obtained by the process of evaporation that has been just described; in less than two minutes its effects were evident; and in five minutes the tint of the paper was changed to a bright brown.

I now conceived that by collecting the water obtained in the second process of slow distillation I should be able to carry on the experiment without *any* appearance of fixed alkali, and the trial proved that I was not mistaken.

Some of this water was introduced into the gold tubes, and the amianthus moistened by it.

After two hours the water in the negative tube produced no effect upon turmeric paper; it did produce an effect upon litmus, which it required great minuteness of observation to perceive; but it wholly lost the power by being heated strongly for two or three minutes, so there is every reason for supposing that it was owing to a small quantity of ammonia.

I made a similar experiment with a portion of the same water in the tubes of agate that had been so often used, and

I had the pleasure of finding the results precisely the same.

To detail any more operations of this kind will be unnecessary; all the facts prove that the fixed alkali is not *generated*, but *evolved*, either from the solid materials employed, or from saline matter in the water.

I have made many experiments in vessels composed of different substances, with the water procured by very slow distillation: and in almost every instance some fixed alkali appeared.

In tubes of wax the alkaline matter was a mixture of soda and potash; and the acid matter a mixture of sulphuric, muriatic, and nitric acids.

In a tube of resin, the alkaline matter seemed to be principally potash.

A cube of Carrara marble of about an inch, having an aperture in its centre, was placed in a crucible of platina, which was filled as high as the upper surface of the cube with the purified water, the aperture was filled with the same fluid; the crucible was positively electrified by a strong VOLTAIC power, and a negatively electrified wire introduced into the aperture.

The water soon gained the property of affecting the tint of turmeric; and fixed alkali and lime were both obtained from it: and this effect took place in repeated experiments: the fixed alkali, however, diminished in quantity every time; and after eleven processes conducted from two to three hours each, disappeared altogether. The production of lime-water was uniform.

I made a solution of 500 grains of this marble in nitric

acid; I decomposed the mixture by carbonate of ammonia, and I collected and evaporated the fluid part, and decomposed the nitrate of ammonia by heat. About $\frac{1}{4}$ of a grain of fixed saline matter remained, which had soda for its base.

It was possible that the Carrara marble might have been recently exposed to sea-water; I therefore tried, in the same way, a piece of granular marble, which I had myself broken from a rock on one of the highest of the primitive mountains of Donegal. It afforded fixed alkali by the agency of negative electricity.

A piece of argillaceous schist from Cornwall, treated in the same manner, gave the same result; and serpentine from the Lizard, and grauwackè from North Wales, both afforded soda. It is probable that there are few stones that do not contain some minute portions of saline matter, which in many cases may be mechanically diffused through their substance: and it is not difficult to conceive the possibility of this, when we consider that all our common rocks and strata bear evident marks of having been anciently covered by the sea.

I was now able to determine distinctly, that the soda procured in glass tubes came principally from the glass, as I had always supposed.

I used the two cones of gold with the purified water and the amianthus; the process was conducted as usual. After a quarter of an hour, the negatively electrified tube did not change the colour of turmeric. I introduced into the top of it a bit of glass; in a few minutes the fluid at the surface rendered the tint of the paper of a deep bright brown.

I had never made any experiments, in which acid matter

having the properties of nitrous acid was not produced, and the longer the operation the greater was the quantity that appeared.

Volatile alkali likewise seemed to be always formed in very minute portions, during the first few minutes in the purified water in the gold cones, but the limit to its quantity was soon attained.

It was natural to account for both these appearances, from the combination of nascent oxygene and hydrogene respectively; with the nitrogene of the common air dissolved in the water: and Dr. PRIESTLEY's experiments on the absorption of gases by water (on this idea) would furnish an easy explanation of the causes of the constant production of the acid, and the limited production of the alkali: for hydrogene, during its solution in water, seems to expel nitrogene; whilst nitrogene and oxygene are capable of co-existing dissolved in that fluid.*

To render the investigation more complete, I introduced the two cones of gold with purified water under the receiver of an air pump; the receiver was exhausted till it contained only $\frac{1}{64}$ of the original quantity of air; and then, by means of a convenient apparatus, the tubes were connected with an active VOLTAIC pile of 50 pairs of plates of 4 inches square. The processs was carried on for 18 hours, when the result was examined. The water in the negative tube produced no effect upon prepared litmus, but that in the positive tube gave it a barely perceptible tinge of red.

An incomparably greater quantity of acid would have been formed in a similar time in the atmosphere, and the small

* PRIESTLEY's Experiments and Observations, Vol. I. page 59.

portion of nitrogene gas remaining in contact with the water, seemed adequate to the effect.

I repeated the experiment under more conclusive circumstances. I arranged the apparatus as before; I exhausted the receiver, and filled it with hydrogene gas from a convenient airholder; I made a second exhaustion, and again introduced hydrogene that had been carefully prepared. The process was conducted for 24 hours, and at the end of this time neither of the portions of the water altered in the slightest degree the tint of litmus.

It seems evident then that water, chemically pure, is decomposed by electricity into gaseous matter alone, into oxygene and hydrogene.

The cause of its decomposition, and of the other decompositions which have been mentioned, will be hereafter discussed.

III. *On the Agencies of Electricity in the Decomposition of various Compounds.*

The experiments that have been detailed on the production of alkali from glass, and on the decomposition of various saline compounds contained in animal and vegetable substances, offered some curious objects of enquiry.

It was evident, that in all changes in which acid and alkaline matter had been present, the acid matter collected in the water round the positively electrified metallic surface; and the alkaline matter round the negatively electrified metallic surface; and this principle of action appeared immediately related to one of the first phænomena observed in the VOLTAIC pile, the

decomposition of the muriate of soda attached to the paste-board; and to many facts which have been since observed on the separation of the constituent parts of neutrosaline and metallic solutions, particularly those detailed by M. M. Hisinger and Berzelius.*

The first experiments that I made immediately with respect to this subject were on the decomposition of solid bodies, insoluble, or difficultly soluble in water. From the effects of the electrical agency on glass, I expected that various earthy compounds would undergo change under similar circumstances; and the results of the trials were decided and satisfactory.

Two cups made of compact sulphate of lime, containing about 14 grain-measures of water each, were connected together by fibrous sulphate of lime, which was moistened by pure water: the cups were filled with this fluid; platina wires from the Voltaic battery of 100 pairs of plates of six inches were introduced into them, so that the circuit of electricity was through the fibrous sulphate of lime. In five minutes the water in the cup connected with the positive wire became acid; that in the opposite cup strongly tinged turmeric. After an hour the fluids were accurately examined; when it was found that a pure and saturated solution of lime had been produced in the cup containing the negative wire, which was partially covered with a crust of lime; and that the other cup was filled with a moderately strong solution of sulphuric acid.

I procured two cubical pieces of crystallized sulphate of strontites, of about an inch; a hole was drilled in each capable of containing about 8 grains of water: the cubes were

* *Annales de Chimie*, Tom. LI. p. 167.

plunged in pure water in a platina crucible; and the level of the fluid preserved a few lines below the surface of the cubes; two platina wires were introduced into the holes, which were filled with pure water. The disengagement of gas, when the wires were connected with the battery of 100, proved that the sulphate of strontites was sufficiently porous to form a proper conducting chain. The results were much longer in being obtained in this experiment than in the last: some time elapsed before a sensible effect could be perceived; but the termination was similar. In 50 hours the fluid in the cavity containing the negative wire had gained the property of precipitating solution of sulphate of potash; and the presence of sulphuric acid in the other cavity was evident from its effect upon solution of muriate of barytes.

I made an experiment upon fluate of lime under like circumsances; but the crystallized fluate not being equally permeable to moisture, the two cavities were connected by moist asbestus. This decomposition was likewise very slow; but in the course of two days a pretty strong solution of lime was obtained in one tube; and an acid fluid in the other, which precipitated acetite of lead, and left a spot upon the glass from which it had been evaporated.

Sulphate of barytes, as might be supposed, proved much more difficult of decomposition than either sulphate of strontites or fluate of lime. I had made four or five experiments upon it, with the same kind of apparatus that had been applied to the fluate of lime, before I was able to gain decided results. In the last process performed on this substance, two pieces of a large single crystal were hollowed by grinding, so as to contain about five grains of water each;

they were connected by moist asbestus, and constantly subjected during four days to the strong action of a battery of 150 pairs of plates of 4 inches square. As the water diminished, its place was supplied by new quantities. At the conclusion of the experiment the fluid on the positive side of the apparatus instantly reddened litmus, tasted very sour, and gave a distinct precipitate with a solution of muriate of barytes; the water on the other side deepened the tincture of turmeric; but did not render solution of sulphate of potash turbid. There was a small quantity of white crust, however, on the sides and the bottom of the cavity, and I conceived that this might be the barytes, which, during the extremely slow decomposition, would have combined with the carbonic acid of the atmosphere. To ascertain if this had been the case, I introduced into the cavity a drop of diluted muriatic acid; a slight effervescence appeared, and the fluid obtained occasioned a distinct white cloudiness in solution of sulphate of soda.

In all these cases the constituent parts of the bodies newly arranged by the effects of electricity existed in considerable quantities, and exposed on a large surface to its action. I had great reason to believe, however, from the trials with distilled water in different vessels, that very minute portions of acid and alkaline matter might be disengaged by this agency from solid combinations, principally consisting of the pure earths.

This part of the investigation was easily elucidated.

For a purpose of geological enquiry, which on a future occasion I shall have the honour of laying before the Society, I had made a careful analysis of a specimen of fine grained basalt from Port Rush in the county of Antrim, by means of

fusion with boracic acid: it had afforded in 100 parts $3\frac{1}{2}$ parts of soda, and nearly $\frac{1}{4}$ a part of muriatic acid, with 15 parts of lime. This stone appeared to me very well fitted for the purpose of experiment: cavities were drilled in two pieces, properly shaped; they contained about 12 grains of water each; they were connected by moistened amianthus, and the process conducted as usual with a power of 50 pairs of plates. At the end of ten hours the result was examined with care. The fluid that had been positively electrified had the strong smell of oxymuriatic acid, and copiously precipitated nitrate of silver; the other portion of fluid affected turmeric, and left by evaporation a substance which seemed to be a mixture of lime and soda.

A part of a specimen of compact zeolite, from the Giant's Causeway, which by analysis had given 7 parts in 100 of soda, had a small cavity made in it; it was immerged in pure water in a crucible of platina, and electrified in the same manner as the cube of Carrara marble, mentioned in page 9. In less than two minutes the water in the cavity had gained the property of changing the colour of turmeric; and in half an hour the solution was disagreeably alkaline to the taste. The matter dissolved proved to be soda and lime.

Lepidolite, treated in the same way, gave potash.

A piece of vitreous lava, from Etna, gave alkaline matter, which seemed to be a mixture of soda, potash, and lime.

As in these trials the object was merely to ascertain the general fact of decomposition, the process was never conducted for a sufficient time to develope a quantity of alkaline matter capable of being conveniently weighed, and of course any loss of weight of the substance could not be determined.

I thought it right, however, to make one experiment of this kind, for the sake of removing every possibility of doubt on the source of the different products; and I selected for this purpose glass, as a substance apparently insoluble in water, and not likely to afford in any way erroneous results.

The balance that I employed was made for the Royal Institution, by Mr. FIDLER, after the model of that belonging to the Royal Society; it turns readily with $\frac{1}{500}$ of a grain when loaded with 100 grains on each side; a glass tube with a platina wire attached, weighing 84 grains $\frac{56}{128}$ was connected with an agate cup, by amianthus; they were filled with purified water, and electrified by a power from 150 pairs of plates, in such a way that the platina in the glass tube was negative. The process was continued for 4 days, when the water was found alkaline. It gave by evaporation and exposure to a heat of about 400° FAHRENHEIT, soda mixed with a white powder insoluble in acids, the whole weight of which was $\frac{36}{128}$ of a grain. The glass tube carefully cleaned and dried weighed 84 grains, $\frac{17}{128}$. The difference between the loss of weight of the tube and the weight of the products in the water may be easily explained: some minute detached particles of amianthus were present, and the soda must have contained water, a substance which it is probably perfectly free from in glass.

Having obtained such results with regard to the disengagement of the saline parts of bodies insoluble in water, I made a number of experiments on soluble compounds; their decomposition was always much more rapid, and the phænomena perfectly distinct.

In these processes I employed the agate cups with platina wires, connected by amianthus moistened in pure water; the

solutions were introduced into the cups, and the electrifying power applied from batteries of 50 pairs of plates, in the usual way.

A diluted solution of sulphate of potash treated in this manner, produced in four hours at the negative wire a weak lixivium of potash; and a solution of sulphuric acid at the positive wire.

The phænomena were similar when sulphate of soda, nitrate of potash, nitrate of barytes, sulphate of ammonia, phosphate of soda, succinate oxalate, and benzoate of ammonia, and alum were used. The acids in a certain time collected in the tube containing the positive wire, and the alkalies and earths in that containing the negative wire.

Solutions of the muriatic salts, decomposed in the same way, uniformly gave oxymuriatic acid on the positive side.

When compatible mixtures of neutrosaline solutions containing the common mineral acids were used, the different acids and the different bases seemed to separate together in a mixed state, without any respect to the orders of affinity.

When metallic solutions were employed, metallic crystals or depositions were formed, as in common GALVANIC experiments, on the negative wire, and oxide was likewise deposited round it; and a great excess of acid was soon found in the opposite cup. With solutions of iron, zinc, and tin, this effect took place, as well as with the more oxidable metals: when muriate of iron was used, the black substance deposited upon the wire was magnetic, and dissolved with effervescence in muriatic acid; and when sulphate of zinc was used, a gray powder possessed of the metallic lustre, and likewise soluble with effervescence, appeared; and in all cases acid in excess was exhibited on the positive side.

Strong or saturated saline solutions, as might have been expected, afforded indications of the progress of decomposition much more rapidly than weak ones; but the smallest proportion of neutrosaline matter seemed to be acted on with energy.

A very simple experiment demonstrates this last principle. If a piece of paper tinged with turmeric, is plunged into pure water in a proper circuit, in contact with the negative point, the very minute quantity of saline compound contained in the paper, affords alkaline matter sufficient to give it instantly a brown tint near its point of contact: and acid in the same manner is immediately developed from litmus paper, at the positive surface.

I made several experiments, with the view of ascertaining whether, in the decompositions by electricity, the separation of the constituent parts was complete from the last portions of the compound; and whenever the results were distinct, this evidently appeared to be the case.

I shall describe one of the most conclusive of the experiments: a very weak solution of sulphate of potash, containing 20 parts water and one part saturated solution, at 64°, was electrified in the two agate cups by the power of 50 pairs of plates for three days: the connecting amianthus, which had been moistened with pure water, was removed, washed with pure water, and again applied, twice every day; by this precaution the presence of any neutral salt that might adhere to it, and disturb the results, was prevented. The alkali obtained in this process in the solution had the properties of pure potash; and when it had been saturated with nitric acid it gave no turbidness by mixture with solution of muriate of

barytes: the acid matter exposed to a strong heat evaporated without leaving any residuum.

IV. *On the Transfer of certain of the constituent Parts of Bodies by the Action of Electricity.*

M. GAUTHEROT has stated,* that in a single GALVANIC circle of zinc, silver, and water, in an active state, the oxide of zinc formed is attracted by the silver;† and M. M. HISINGER and BERZELIUS detail an account of an experiment, in which solution of muriate of lime being placed in the positive part of a siphon, electrified by wires from a VOLTAIC pile, and distilled water in the negative part, lime appeared in the distilled water.

These facts rendered it probable, that the saline elements evolved in decompositions by electricity were capable of being transferred from one electrified surface to another, according to their usual order of arrangement; but to demonstrate this clearly, new researches were wanting.

I connected one of the cups of sulphate of lime, mentioned page 13, with a cup of agate by asbestus; and, filling them with purified water, made the platina wire in the cup of sulphate of lime transmit the electricity from a power of 100; a wire in the agate cup received it. In about four hours a strong solution of lime was found in the agate cup, and sulphuric acid in the cup of sulphate of lime. By reversing the order, and carrying on the process for a similar time, the sulphuric acid appeared in the agate cup, and the solution of lime on the opposite side.

Many trials were made with other saline substances, with

* *Annales de Chimie*, Vol. XXXIX. page 203. † Ibid. Vol. LI. page 171.

analogous results. When the compounds of the strong mineral acids with alkaline or alkaline-earthy bases, were introduced into one tube of glass, distilled water connected by amianthus being in another tube, both connected by wires of platina in the VOLTAIC arrangement, the base always passed into the distilled water when it was negative, and the acid when it was positive.

The metals and the metallic oxides passed towards the negative surface like the alkalies, and collected round it. In a case in which solution of nitrate of silver was used on the positive side, and distilled water on the negative, silver appeared on the whole of the transmitting amianthus, so as to cover it with a thin metallic film.

The time required for these transmissions (the quantity and intensity of the electricity, and other circumstances remaining the same) seemed to be in some proportion as the length of the intermediate volume of water. Thus when with the power of 100, sulphate of potash was on the negative side, and distilled water on the positive side, the distance between the wires being only an inch, sulphuric acid, in sufficient quantity to be very manifest, was found in the water in less than five minutes: but when the tubes were connected by an intermediate vessel of pure water, so as to make the circuit 8 inches, 14 hours were required to produce the same effect.

To ascertain whether the contact of the saline solution with a metallic surface was necessary for the decomposition and transfer, I introduced purified water into two glass tubes; a vessel containing solution of muriate of potash was connected with them respectively by amianthus; and the arrangement was made in such a way, that the level of both the portions

of purified water was higher than the level of the saline solution.

In this case, the saline matter was distant from each of the wires at least $\frac{3}{5}$ of an inch; yet alkaline matter soon appeared in one tube, and acid matter in the other: and in 16 hours moderately strong solutions of potash, and of muriatic acid had been formed.

In this case of electrical transfer or attraction, the acid and alkaline matter seemed to be perfectly pure; and I am inclined to believe that this is uniformly the case in all experiments carefully made. One of the instances in which I conceived acid most likely to be present, was in the transfer of magnesia from sulphate of magnesia in the positive tube, to distilled water in the negative tube. I examined the case, taking care that the distilled water was never upon a lower level than the saline solution: the process was continued for some hours, till a considerable quantity of magnesia had appeared. The connecting amianthus was removed, and muriatic acid poured into the tube: the saturated solution did not precipitate solution of muriate of barytes.

I endeavoured to ascertain the progress of the transfer, and the course of the acid or alkaline matter in these decompositions, by using solutions of litmus and turmeric, and papers coloured by these substances; and these trials led to the knowledge of some singular and unexpected circumstances.

Two tubes, one containing distilled water, the other solution of sulphate of potash, were each connected by amianthus with a small *oz.* measure filled with distilled water tinged by litmus: the saline solution was negatively electrified; and as it was natural to suppose, that the sulphuric acid in passing

through the water to the positive side would redden the litmus in its course, some slips of moistened paper tinged with litmus were placed above and below the pieces of amianthus, directly in the circuit. The progress of the experiment was minutely observed; the first effect of reddening took place immediately above the positive surface, where I had least expected it; the red tint slowly diffused itself from the positive side to the middle of the vessel, but no redness appeared above the amianthus, or about it, on the negative side, and though it had been constantly transmitting sulphuric acid, it remained unaffected to the last.

The order of the experiment was changed, and the saline solution placed on the positive side; a solution and papers tinged with turmeric being substituted for those tinged with litmus. The effect was precisely analogous; the turmeric became brown first, near the negative wire, and no change took place in the intermediate vessel near the positive wire.

In another process, the two glass tubes were filled with solution of muriate of soda, and the intermediate vessel with solution of sulphate of silver; paper tinged with turmeric was placed on the positive side, and paper tinged with litmus on the negative side; as soon as the electrical circuit was complete, soda began to appear in the negative tube, and oxymuriatic acid in the positive tube; and the alternate products were exhibited passing into the solution of sulphate of silver, the muriatic acid occasioning a dense heavy precipitate, and the soda a more diffused and a lighter one; but neither the turmeric transmitting the alkali, nor the litmus transmitting the acid, had their tints in the slightest degree altered.

V. *On the Passage of Acids, Alkalies, and other Substances through various attracting chemical Menstrua, by Means of Electricity.*

As acid and alkaline substances during the time of their electrical transfer passed through water containing vegetable colours without affecting them, or apparently combining with them, it immediately became an object of enquiry, whether they would not likewise pass through chemical menstrua, having stronger attractions for them; and it seemed reasonable to suppose, that the same power which destroyed elective affinity in the vicinity of the metallic points, would likewise destroy it, or suspend its operation, throughout the whole of the circuit.

An arrangement was made, of the same vessels and apparatus employed in the experiment on the solution of muriate of soda and sulphate of silver, page 23. Solution of sulphate of potash was placed in contact with the negatively electrified point, pure water was placed in contact with the positively electrified point, and a weak solution of ammonia was made the middle link of the conducting chain; so that no sulphuric acid could pass to the positive point in the distilled water, without passing through the solution of ammonia.

The power of 150 was used: in less than five minutes it was found, by means of litmus paper, that acid was collecting round the positive point; in half an hour, the result was sufficiently distinct for accurate examination.

The water was sour to the taste, and precipitated solution of nitrate of barytes.

Similar experiments were made with solution of lime; and

weak solutions of potash and soda, and the results were analogous. With strong solutions of potash and soda a much longer time was required for the exhibition of the acid; but even with the most saturated alkaline lixivium, it always appeared in a certain period.

Muriatic acid, from muriate of soda, and nitric acid from nitrate of potash, were transmitted through concentrated alkaline menstrua, under similar circumstances.

When distilled water was placed in the negative part of the circuit, and a solution of sulphuric, muriatic, or nitric acid, in the middle, and any neutral salt with a base of lime, soda, potash, ammonia, or magnesia, in the positive part, the alkaline matter was transmitted through the acid matter to the negative surface, with similar circumstances to those occurring during the passage of the acid through the alkaline menstrua; and the less concentrated the solution, the greater seemed to be the facility of transmission.

I tried in this way muriate of lime with sulphuric acid, nitrate of potash with muriatic acid, sulphate of soda with muriatic acid, and muriate of magnesia with sulphuric acid; I employed the power of 150; and in less than 48 hours, I gained in all these cases decided results; and magnesia came over like the rest.

Strontites and barytes passed, like the other alkaline substances, readily through muriatic and nitric acids; and, *vice versâ*, these acids passed with facility through aqueous solutions of barytes and strontites; but in experiments in which it was attempted to pass sulphuric acid through the *same menstrua*, or to pass barytes or strontites through this acid, the results were very different.

When solution of sulphate of potash was in the negative part of the circuit, distilled water in the positive part, and saturated solution of barytes in the middle, no sensible quantity of sulphuric acid existed in the distilled water after 30 hours, the power of 150 being used; after four days, sulphuric acid appeared, but the quantity was extremely minute; much sulphate of barytes had formed in the intermediate vessel; the solution of barytes was so weak as barely to tinge litmus; and a thick film of carbonate of barytes had formed on the surface of the fluid. With solution of strontites the result was very analogous, but the sulphuric acid was sensible in three days.

When solution of muriate of barytes was made positive by the power of 150, concentrated sulphuric acid intermediate, and distilled water negative. No barytes appeared in the distilled water, when the experiment had been carried on for four days; but much oxymuriatic acid had formed in the positive vessel, and much sulphate of barytes had been deposited in the sulphuric acid.

Such of the metallic oxides as were made subjects of experiment passed through acid solutions from the positive to the negative side, but the effect was much longer in taking place than in the instances of the transition of alkaline matter. When solution of green sulphate of iron was made positive, solution of muriatic acid intermediate, and water negative, in the usual arrangement, green oxide of iron began to appear in about ten hours upon the negative connecting amianthus, and in three days a considerable portion had been deposited in the tube. Analogous results were obtained with sulphate of copper, nitrate of lead, and nitromuriate of tin.

I made several experiments on the transition of alkaline and acid matter through different neutrosaline solutions, and the results were such as might well have been anticipated.

When solution of muriate of barytes was negative, solution of sulphate of potash intermediate, and pure water positive, the power being from 150, sulphuric acid appeared in about five minutes in the distilled water; and in two hours the muriatic acid was likewise very evident. When solution of sulphate of potash was positive, solution of muriate of barytes intermediate, and distilled water negative, the barytes appeared in the water in a few minutes; the potash from the more remote part of the chain was nearly an hour in accumulating, so as to be sensible.

When the solution of muriate of barytes was positive, the solution of sulphate of potash intermediate, and distilled water negative, the potash soon appeared in the distilled water; a copious precipitation of sulphate of barytes formed in the middle vessel; but after ten hours no barytes had passed into the water.

When solution of sulphate of silver was interposed between solution of muriate of barytes on the negative side, and pure water on the positive side, sulphuric acid alone passed into the distilled water; and there was a copious precipitation in the solution of sulphate of silver. This process was carried on for ten hours.

I tried several of these experiments of transition upon vegetable and animal substances with perfect success.

The saline matter exposed in contact with the metal, and that existing in the vegetable or animal substances, both underwent decomposition and transfer; and the time of the appearance of the different products at the extremities of the circuit was governed by the degree of their vicinity.

Thus, when a fresh leaf-stalk of the polyanthus, about 2 inches long, was made to connect a positively electrified tube containing solution of nitrate of strontites, and a negatively electrified tube containing pure water. The water soon became green, and gave indications of alkaline properties, and free nitric acid was rapidly separated in the positive tube. After ten minutes, the alkaline matter was examined; it consisted of potash and lime, and as yet no strontites had been carried into it: for the precipitate it gave with sulphuric acid readily dissolved in muriatic acid. In half an hour strontites, however, appeared; and in four hours it formed a very abundant ingredient of the solution.

A piece of muscular flesh of beef, of about 3 inches in length and half an inch in thickness, was treated in the same way as the medium of communication between muriate of barytes and distilled water. The first products were soda, ammonia, and lime; and after an hour and a quarter, the barytes was very evident. There was much free oxymuriatic acid in the positively electrified tube, but no particle of muriatic acid had passed into the negative tube, either from the muriatic solution or from the muscular fibre.

VI. *Some general Observations on these Phænomena, and on the Mode of Decomposition and Transition.*

It will be a general expression of the facts that have been detailed, relating to the changes and transitions by electricity, in common philosophical language, to say, that hydrogene, the alkaline substances, the metals, and certain metallic oxides, are attracted by negatively electrified metallic surfaces, and repelled by positively electrified metallic surfaces; and con-

trariwise, that oxygene and acid substances are attracted by positively electrified metallic surfaces, and repelled by negatively electrified metallic surfaces; and these attractive and repulsive forces are sufficiently energetic to destroy or suspend the usual operation of elective affinity.

It is very natural to suppose, that the repellent and attractive energies are communicated from one *particle to another particle* of the same kind, so as to establish a conducting chain in the fluid; and that the locomotion takes place in consequence; and that this is really the case seems to be shown by many facts. Thus, in all the instances in which I examined alkaline solutions through which acids had been transmitted, I always found acid in them whenever any acid matter remained at the original source. In time, by the attractive power of the positive surface, the decomposition and transfer undoubtedly become complete; but this does not affect the conclusion.

In the cases of the separation of the constituents of water, and of solutions of neutral salts forming the whole of the chain, there may possibly be a succession of decompositions and recompositions throughout the fluid. And this idea is strengthened by the experiments on the attempt to pass barytes through sulphuric acid, and muriatic acid through solution of sulphate of silver, in which, as insoluble compounds are formed and carried out of the sphere of the electrical action, the power of transfer is destroyed. A similar conclusion might likewise be drawn from many other instances. Magnesia and the metallic oxides, as I have already mentioned, will pass along moist amianthus from the positive to the negative surface; but if a vessel of pure water be interposed, they do not reach the negative vessel, but sink to the bottom.

These experiments I have very often made, and the results are perfectly conclusive; and in the case, page 26, in which sulphuric acid seemed to pass in small quantities through very weak solutions of strontites and barytes, I have no doubt but that it was carried through by means of a thin stratum of pure water, where the solution had been decomposed at the surface by carbonic acid; for in an experiment similar to these in which the film of carbonate of barytes was often removed and the fluid agitated, no particle of sulphuric acid appeared in the positive part of the chain.

It is easy to explain, from the general phænomena of decomposition and transfer, the mode in which *oxygene* and *hydrogene* are separately evolved from water. The oxygene of a portion of water is attracted by the positive surface, at the same time that the other constituent part, the hydrogene is repelled by it; and the opposite process takes place at the negative surface; and in the middle or neutral point of the circuit, whether there be a series of decompositions and recompositions, or whether the particles from the extreme points only are active, there must be a new combination of the repelled matter: and the case is analogous to that of two portions of muriate of soda separated by distilled water; muriatic acid is repelled from the negative side, and soda from the positive side, and muriate of soda is composed in the middle vessel.

These facts seem fully to invalidate the conjectures of M. RITTER, and some other philosophers, with regard to the elementary nature of water, and perfectly to confirm the great discovery of Mr. CAVENDISH.

M. RITTER conceived that he had procured oxygene from water without hydrogene, by making sulphuric acid the medium of communication at the negative surface; but in this case,

sulphur is deposited, and the oxygene from the acid, and the hydrogene from the water are respectively repelled; and a new combination produced.

I have attempted some of the experiments of decomposition and transfer, by means of common electricity, making use of a powerful electrical machine of Mr. NAIRNE's construction, belonging to the Royal Institution, of which the cylinder is 15 inches in diameter, and 2 feet long.

With the same apparatus as that employed for decompositions by the VOLTAIC battery, no perceptible effect was produced by passing a strong current of electricity silently for four hours through solution of sulphate of potash.

But by employing fine platina points of $\frac{1}{70}$ of an inch in diameter, cemented in glass tubes in the manner contrived by Dr. WOLLASTON,* and bringing them near each other, in vessels containing from 8 to 4 grains of the solution, and connected by moist asbestus, potash appeared in less than two hours round the negatively electrified point, and sulphuric acid round the positive point.

In a similar experiment sulphuric acid was transferred through moist asbestus into water; so that there can be no doubt, that the principle of action is the same in common and the VOLTAIC electricity.†

* Phil. Trans. Vol. XCI. page 427.

† This had been shewn, with regard to the decomposition of water, by Dr. WOLLASTON's important researches. — By carefully avoiding sparks, I have been able to obtain the two constituents in a separate state. In an experiment in which a fine platina point cemented in glass, and connected by a single wire with the positive conductor of this machine, was plunged in distilled water in an insulated state, and the electricity dissipated into the atmosphere by means of moistened filaments of cotton, oxygene gas, mixed with a little nitrogene gas, was produced; and when the same apparatus was applied to the negative conductor hydrogene gas was evolved, and a minute

VII. *On the general Principles of the chemical Changes produced by Electricity*

The experiments of Mr. BENNET had shown, that many bodies brought into contact and afterwards separated, exhibited *opposite* states of electricity; but it is to the investigations of VOLTA that a clear developement of the fact is owing; he has distinctly shown it in the case of copper and zinc, and other metallic combinations; and has supposed that it also takes place with regard to metals and fluids.

In a series of experiments made in 1801,* on the construction of electrical combinations by means of alternations of single metallic plates, and different strata of fluids, I observed that when acid and alkaline solutions were employed as elements of these instruments, the alkaline solutions always received the electricity from the metal, and the acid always transmitted it to the metal; thus, in an arrangement of which the elements were tin, water, and solution of potash, the circulation of the electricity was from the water to the tin, and from the tin to the solution of potash; but in an arrangement composed of weak nitric acid, water, and tin; the order was from the acid to the tin, and from the tin to the water.

These principles seem to bear an immediate relation to the general phænomena of decomposition and transference, which have been the subject of the preceding details.

portion of oxygene and nitrogene gases: but neither of the foreign products, the nitrogene gas in the one case and the nitrogene and oxygene gases in the other, formed as much as $\frac{1}{10}$ part of the volume of the gases; and there is every reason to suppose that they were derived from the extrication of common air, which had been dissolved in the water. This result, which when I first obtained it in 1803, appeared very obscure, is now easily explained; the alternate products must have been evolved at the points of the dissipation of the electricity.

* See Phil. Trans. Vol. XCI. page 397.

In the simplest case of electrical action, the alkali which receives electricity from the metal would necessarily, on being separated from it, appear positive; whilst the acid under similar circumstances would be negative; and these bodies having respectively with regard to the metals, that which may be called a positive and a negative electrical energy, in their repellent and attractive functions seem to be governed by laws the same as the common laws of electrical attraction and repulsion. The body possessing the positive energy being repelled by positively electrified surfaces, and attracted by negatively electrical surfaces; and the body possessing the negative energy following the contrary order.

I have made a number of experiments with the view of elucidating this idea, and of extending its application; and in all cases they have tended to confirm the analogy in a remarkable manner.

Well burned charcoal water and nitric acid, the same substance water and solution of soda, made respectively elements of different electrical combinations, became distinctly active when 20 alternations were put together: the positive energy being exhibited on the side of the alkali, and the negative on that of the acid. Arrangements of plates of zinc, pieces of moistened pasteboard, and moistened quicklime, to the number of 40 series, likewise formed a weak electrical pile, the effect of the lime being similar to that of an alkali, but the power was soon lost.

I endeavoured, by means of very delicate instruments, to ascertain the electrical states of single insulated acid and alkaline solutions, after their contact with metals; and for this purpose I employed at different times the condensing

electrometer of Mr. CUTHBERTSON's construction, Mr. CAVALLO's multiplier, and a very sensible electrical balance, on the principle of torsion, adopted by M. COULOMB; but the effects were unsatisfactory, the circumstances of evaporation, and of chemical action, and the adherence of the solutions to the surfaces of the metals employed, in most cases, prevented any distinct result, or rendered the source of the electricity doubtful. I shall not enter into any details of these processes, or attempt to draw conclusions from capricious and uncertain appearances, which, as we shall immediately see, may be fully deduced from clear and distinct ones.

The alkaline and acid substances capable of existing in the dry and solid form, give by contact with the metals exceedingly sensible electricities, which require for their exhibition the gold leaf electrometer only with the small condensing plate.

When oxalic, succinic, benzoic or boracic acid, perfectly dry, either in powder or crystals, were touched upon an extended surface with a plate of copper insulated by a glass handle, the copper was found positive, the acid negative. In favourable weather, and when the electrometer was in perfect condition, one contact of the metal was sufficient to produce a sensible charge; but seldom more than five or six were required. Other metals, zinc, and tin for instance were tried with the same effect. And the metal received the positive charge, apparently to the same extent, whether the acid was insulated upon glass, or connected with the ground.

The solid acid of phosphorus, which had been strongly ignited, and most carefully excluded from the contact of air, rendered the insulated plate of zinc positive by four contacts;

but after exposure to the atmosphere for a few minutes it wholly lost this power.

When metallic plates were made to touch dry lime, strontites, or magnesia, the metal became negative; the effect was exceedingly distinct, a single contact upon a large surface being sufficient to communicate a considerable charge. For these experiments the earths were carefully prepared; they were in powder, and had been kept for several days in glass bottles before they were used: it is essential to the success of the process that they be of the temperature of the atmosphere. In some experiments which I made upon them when cooling, after having been ignited; they appeared strongly electrical, and rendered the conductors brought in contact with them positive.

I made several experiments in a similar manner on the effects of the contact of potash and soda with the metals. Potash in no instance afforded a satisfactory result; its powerful attraction for water presents an obstacle probably unsurmountable to the success of any trials made in the free atmosphere. Soda, in the only case in which electricity was exhibited, affected the metal in the same way as lime, strontites, and magnesia. Upon this occasion the soda had been prepared with great care, exposed in a platina crucible for nearly an hour in a red heat, and suffered to cool in the crucible inverted over mercury; when cool it was immediately removed, and the contact made with a plate of zinc: the experiment was performed in the open air; the weather was peculiarly dry, the thermometer stood at 28° FAHRENHEIT, and the barometer at 30.2 inches; six contacts gave a charge to the condensing electrometer in the first trial; in the second ten were required to produce a similar effect; and after this,

though two minutes only had elapsed, no farther result could be obtained.

In the decomposition of sulphuric acid by VOLTAIC electricity the sulphur separates on the negative side. The experiments of various electricians prove, that by the friction of sulphur and metals, the sulphur becomes positive and the metals negative; the same thing I find happens from the contact of an unexited cake of sulphur and insulated metallic plates. Mr. WILKE has stated an exception to lead, as rendering sulphur negative by its friction. The results that I have obtained with lead, in trials very carefully made, are the same as those with other metals.* Sulphur, by being rubbed or struck against newly-polished lead, always became positive. Mr. WILKE perhaps was misled by using tarnished lead: sulphur, I find, rubbed against litharge, or lead the surface of which has been long exposed to air, becomes negative; and this exception being removed, all the facts on the subject are confirmations of the general principle.†

* As sulphur is a nonconductor, and easily excited by slight friction, or small changes in its temperature, some caution is required in drawing conclusions from the experiments in which it is employed. Sulphur, examined immediately after having been heated, gives a positive charge to conductors, agreeing in this respect with the alkaline substances; and a slight contact with the dry hand is sufficient to render it negative. In general likewise in experiments of contact care should be taken that the metallic plate is free from electricity: well polished plates of copper and zinc will, I find, receive a negative charge from being laid on a table of common mahogany.

† Concentrated solution of phosphoric acid, I find, is decomposed by VOLTAIC electricity: the phosphorus combines with the negatively electrified metal, and forms a phosphuret; at least this happened in the two cases that I tried with platina and copper. From all analogy it may be inferred, that the electrical energy of this inflammable substance with regard to metals is the same as that of sulphur. I tried some experiments of contact upon it, but without success. Its slow combustion in the atmosphere it is most likely was the cause of the failure: but even in gases not containing free or loosely combined oxygene, its evaporation would probably interfere.

On the general principle, oxygene and hydrogene ought to possess, with regard to the metals respectively, the negative and positive energy. This I have not been able to prove by direct experiments of contact; but the idea is confirmed by the agency of their compounds; thus I have found that solution of sulphuretted hydrogene in water acts in the electrical apparatus composed of single plates and different strata of fluids, in the same manner as alkaline solutions; and that solution of oxymuriatic acid is more powerful in similar arrangements than solutions of muriatic acid of a higher degree of concentration; and in both these cases, it is impossible to conceive the combined hydrogene and oxygene inactive. The inference likewise is fully warranted by the case of the solutions of alkaline hydroguretted sulphurets, which consisting principally of alkali and sulphur together in union with water, exhibit the positive energy with regard to the metals in a very high degree. In the series of experiments on VOLTAIC arrangements constructed with single plates abovementioned, I found the solutions of hydroguretted sulphurets in general much more active than alkaline solutions, and particularly active with copper, silver, and lead. And in an experiment that I made on a combination of copper, iron, and hydroguretted sulphurets of potash, in 1802, I found that the positive energy of the hydroguretted sulphurets with regard to the copper was sufficient to overpower that of the iron; so that the electricity did not circulate from the copper to the iron, and from the iron to the fluid, as in common cases, but from the copper to the hydroguretted sulphuret, and from the hydroguretted sulphuret to the iron.

All these details afford the strongest confirmation of the

principle. It may be considered almost as a mere arrangement of facts; and with some extensions it seems capable of being generally applied.

Bodies possessing opposite electrical energies with regard to one and the same body, we might fairly conclude would likewise possess them with regard to each other. This I have found by experiment is the case with lime and oxalic acid. A dry piece of lime, made from a very pure compact secondary limestone, and of such a form as to present a large smooth surface, became positively electrical by repeated contacts with crystals of oxalic acid: and these crystals placed upon the top of a condensing electrometer, and repeatedly touched by the lime, which after each contact was freed from its charge, rendered the gold leaves negatively electrical. The tendency of the mere contacts of the acid and alkali with the metal would be to produce opposite effects to those exhibited, so that their mutual agency must have been very energetic.

It will not certainly be a remote analogy to consider the other acid and alkaline substances generally, and oxygene and hydrogene as possessing similar electrical relations; and in the decompositions and changes presented by the effects of electricity, the different bodies naturally possessed of chemical affinities appear incapable of combining, or of remaining in combination, when placed in a state of electricity different from their natural order. Thus, as we have seen, the acids in the positive part of the circuit, separate themselves from alkalies, oxygene from hydrogene, and so on; and metals on the negative side, do not unite to oxygene, and acids do not remain in union with their oxides; and in this way the attrac-

tive and repellent agencies seem to be communicated from the metallic surfaces throughout the whole of the menstruum.

VIII. *On the Relations between the electrical Energies of Bodies, and their chemical Affinities.*

As the chemical attraction between two bodies seems to be destroyed by giving one of them an electrical state different from that which it naturally possesses; that is, by bringing it artificially into a state similar to the other, so it may be increased by exalting its natural energy. Thus, whilst zinc, one of the most oxidable of the metals, is incapable of combining with oxygene when negatively electrified in the circuit, even by a feeble power; silver, one of the least oxidable, easily unites to it when positively electrified; and the same thing might be said of other metals.

Amongst the substances that combine chemically, all those, the electrical energies of which are well known, exhibit opposite states; thus, copper and zinc, gold and quicksilver, sulphur and the metals, the acid and alkaline substances, afford apposite instances; and supposing perfect freedom of motion in their particles or elementary matter, they ought, according to the principles laid down, to attract each other in consequence of their electrical powers. In the present state of our knowledge, it would be useless to attempt to speculate on the remote cause of the electrical energy, or the reason why different bodies, after being brought into contact, should be found differently electrified; its relation to chemical affinity is, however, sufficiently evident. May it not be identical with it, and an essential property of matter?

The coated glass plates of BECCARIA strongly adhere to each other when oppositely charged, and retain their charges on being separated. This fact affords a distinct analogy to the subject; different particles in combining must still be supposed to preserve their peculiar states of energy.

In the present early stage of the investigation, it would be improper to place unbounded confidence in this hypothesis; but it seems naturally to arise from the facts, and to coincide with the laws of affinity, so ably developed by modern chemists; and the general application of it may be easily made.

Supposing two bodies, the particles of which are in different electrical states, and those states sufficiently exalted to give them an attractive force superior to the power of aggregation, a combination would take place which would be more or less intense according as the energies were more or less perfectly balanced; and the change of properties would be correspondently proportional.

This would be the simplest case of chemical union. But different substances have different degrees of the same electrical energy in relation to the same body: thus the different acids and alkalies are possessed of different energies with regard to the same metal; sulphuric acid, for instance, is more powerful with lead than muriatic acid, and solution of potash is more active with tin than solution of soda. Such bodies likewise may be in the same state or repellent with regard to each other, as apparently happens in the cases just mentioned; or they may be neutral; or they may be in opposite or attracting states, which last seems to be the condition of sulphur and alkalies that have the same kind of energy with regard to metals.

When two bodies repellent of each other act upon the same body with different degrees of the same electrical attracting energy, the combination would be determined by the degree; and the substance possessing the weakest energy would be repelled; and this principle would afford an expression of the causes of elective affinity, and the decompositions produced in consequence.

Or where the bodies having different degrees of the same energy, with regard to the third body, had likewise different energies with regard to each other, there might be such a balance of attractive and repellent powers as to produce a triple compound; and by the extension of this reasoning, complicated chemical union may be easily explained.

Numerical illustrations of these notions might be made without difficulty, and they might be applied to all cases of chemical action; but in the present state of the enquiry, a great extension of this hypothetical part of the subject would be premature.

The general idea will, however, afford an easy explanation of the influence of affinity by the masses of the acting substances, as elucidated by the experiments of M. BERTHOLLET; for the combined effect of many particles possessing a feeble electrical energy, may be conceived equal or even superior to the effect of a few particles possessing a strong electrical energy: and the facts mentioned, page 25, confirm the supposition: for concentrated alkaline lixivia resist the transmission of acids by electricity much more powerfully than weak ones.

Allowing combination to depend upon the balance of the natural electrical energies of bodies, it is easy to conceive that

a *measure* may be found of the artificial energies, as to intensity and quantity produced in the common electrical machine, or the VOLTAIC apparatus, capable of destroying this equilibrium; and such a measure would enable us to make a scale of electrical powers corresponding to degrees of affinity.

In the circuit of the VOLTAIC apparatus, completed by metallic wires and water, the strength of the opposite electricities diminish from the points of contact of the wires towards the middle point in the water, which is necessarily neutral. In a body of water of considerable length it probably would not be difficult to assign the places in which the different neutral compounds yielded to, or resisted, decomposition. Sulphate of barytes, in all cases that I tried, required immediate contact with the wire: solution of sulphate of potash exhibited no marks of decomposition with the power of 150, when conconnected in a circuit of water ten inches in length, at four inches from the positive point; but when placed within two inches, its alkali was slowly repelled and its acid attracted.*

Whenever bodies brought by artificial means into a high state of opposite electricities are made to restore the equilibrium, heat and light are the common consequences. It is

* In this experiment, the water was contained in a circular glass bason two inches deep, the communication was made by pieces of amianthus of about the eighth of an inch in breadth. The saline solution filled a half ounce measure, and the distance between the solution and the water, at both points of communication, was a quarter of an inch. I mention these circumstances because the quantity of fluid and the extent of surface materially influence the result in trials of this kind. Water included in glass siphons forms a much less perfect conducting chain than when diffused upon the surface of fibrous nonconducting substances of much smaller volume than the diameter of the siphons. I attempted to employ siphons in some of my first experiments; but the very great inferiority of effect as compared with that of amianthus made me altogether relinquish the use of them.

perhaps an additional circumstance, in favour of the theory to state, that heat and light are likewise the result of all intense chemical action. And as in certain forms of the VOLTAIC battery, where large quantities of electricity of low intensity act, heat is produced without light; so in slow combinations there is an increase of temperature without luminous appearance.

The *effect of* HEAT, in producing combination, may be easily explained according to these ideas. It not only often gives more freedom of motion to the particles, but in a number of cases it seems to exalt the electrical energies of bodies; glass, the tourmalin, sulphur, all afford familiar instances of this last species of energy.

I heated together an insulated plate of copper and a plate of sulphur, and examined their electricities as their temperature became elevated: these electricities, scarcely sensible at 56° FAHRENHEIT to the condensing electrometer, became at 100° FAHRENHEIT capable of affecting the gold leaves without condensation; they increased in a still higher ratio as the sulphur approached towards its point of fusion. At a little above this point, as is well known from the experiments of the Dutch chemists, the two substances rapidly combine, and heat and light are evident.

Similar effects may be conceived to occur in the case of oxygene and hydrogene, which form water, a body apparently neutral in electrical energy to most other substances: and we may reasonably conclude that there is the same exaltation of power, in all cases of combustion. In general, when the different energies are strong and in perfect equilibrium, the combination ought to be quick, the heat and light intense, and

the new compound in a neutral state. This would seem to be the case in the instance just quoted; and in the circumstances of the union of the strong alkalies and acids. But where one energy is feeble and the other strong, all the effects must be less vivid; and the compound, instead of being neutral, ought to exhibit the excess of the stronger energy.

This last idea is confirmed by all the experiments which I have been able to make on the energies of the saline compounds with regard to the metals. Nitrate and sulphate of potash, muriate of lime, oxymuriate of potash, though repeatedly touched upon a large surface by plates of copper and zinc, gave no electrical charge to them; subcarbonate of soda and borax, on the contrary, gave a slight negative charge, and alum and superphosphate of lime a feeble positive charge.

Should this principle on further enquiry be found to apply generally, the degree of the electrical energies of bodies, ascertained by means of sensible instruments, will afford new and useful indications of their composition.

IX. *On the Mode of Action on the Pile of* VOLTA, *with experimental Elucidations.*

The great tendency of the attraction of the different chemical agents, by the positive and negative surfaces in the VOLTAIC apparatus, seems to be to restore the electrical equilibrium. In a VOLTAIC battery, composed of copper, zinc, and solution of muriate of soda, all circulation of the electricity ceases, the equilibrium is restored if copper be brought in contact with the zinc on both sides: and oxygene and acids,

which are attracted by the positively electrified zinc, exert similar agencies to the copper, but probably in a slighter degree, and being capable of combination with the metal, they produce a momentary equilibrium only.

The electrical energies of the metals with regard to each other, or the substances dissolved in the water, in the VOLTAIC and other analogous instruments, seem to be the causes that disturb the equilibrium, and the chemical changes the causes that tend to restore the equilibrium; and the phænomena most probably depend on their joint agency.

In the VOLTAIC pile of zinc, copper, and solution of muriate of soda, in what has been called its condition of electrical tension, the communicating plates of copper and zinc are in opposite electrical states. And with regard to electricities of such very low intensity, water is an insulating body: every copper-plate consequently produces by induction an increase of positive electricity upon the opposite zinc plate; and every zinc plate an increase of negative electricity on the opposite copper-plate: and the intensity increases with the number, and the quantity with the extent of the series.

When a communication is made between the two extreme points, the opposite electricities tend to annihilate each other; and if the fluid medium could be a substance incapable of decomposition, the equilibrium, there is every reason to believe, would be restored, and the motion of the electricity cease. But solution of muriate of soda being composed of two series of elements possessing opposite electrical energies, the oxygene and the acid are attracted by the zinc, and the hydrogene and the alkali by the copper. The balance of power is momentary only; for solution of zinc is formed, and the hydro-

gene disengaged. The negative energy of the copper and the positive energy of the zinc are consequently again exerted, enfeebled only by the opposing energy of the soda in contact with the copper, and the process of electromotion continues, as long as the chemical changes are capable of being carried on.

This theory in some measure reconciles the hypothetical principles of the action of the pile adopted by its illustrious inventor, with the opinions concerning the chemical origin of GALVANISM, supported by the greater number of the British philosophers, and it is confirmed and strengthed by many facts and experiments.

Thus the VOLTAIC pile of 20 pairs of plates of copper and zinc exhibits no permanent electromotive power when the connecting fluid is water free from air;* for this substance does not readily undergo chemical change, and the equilibrium seems to be capable of being permanently restored through it. Concentrated sulphuric acid, which is a much more perfect conductor, is equally inefficient, for it has little action upon zinc, and is itself decomposed only by a very strong power. Piles, containing as their fluid element either pure water or sulphuric acid, will undoubtedly give single shocks, and this effect is connected with the restoration of the equilibrium disturbed by the energies of the metals; but when their extreme plates are connected there is no exhibition, as in usual cases of electromotion. Water containing loosely combined oxygene is more efficient than water containing common air, as it enables

* The experiments proving this fact, and the other analogous facts in this page, may be seen detailed in Nicholson's Journal, 4to. Vol. IV. page 338 and 394; and Phil. Mag. Vol. X. page 40.

oxide of zinc to be formed more rapidly, and in larger quantities. Neutrosaline solutions which are at first very active, loose their energy in proportion as their acid arranges itself on the side of the zinc, and their alkali on that of the copper; and I have found the powers of a combination nearly destroyed from this cause very much revived, merely by agitating the fluids in the cells and mixing their parts together. Diluted acids, which are themselves easily decomposed, or which assist the decomposition of water, are above all other substances powerful; for they dissolve the zinc, and furnish only a gaseous product to the negative surface, which is immediately disengaged.

There are other experiments connected with very striking results, which offer additional reasons for supposing the decomposition of the chemical menstrua essential to the continued electromotion in the pile.

As when an electrical discharge is produced by means of small metallic surfaces in the VOLTAIC battery, (the opposite states being exalted,) *sensible* heat is the consequence, it occurred to me, that if the decomposition of the chemical agents was essential to the balance of the opposed electricities, the effect, in a saline solution, of this decomposition, and of the transfer of the alkali to the negative side, and of the acid to the positive side, ought, under favourable circumstances, to be connected with an increase of temperature.

I placed the gold cones, which have been so often mentioned, in the circuit of the battery with the power of 100, I filled them with distilled water, and connected them by a piece of moistened asbestus, about an inch in length and $\frac{1}{6}$ of an inch diameter; I provided a small air-thermometer capable

of being immersed in the gold cones, expecting (if any) only a very slight change of temperature; I introduced a drop of solution of sulphate of potash into the positive cone: the decomposition instantly began: potash passed rapidly over into the negative cone, heat was immediately sensible; and in less than two minutes the water was in a state of ebullition.

I tried the same thing with a solution of nitrate of ammonia, and in this instance the heat rose to such an intensity as to evaporate all the water in three or four minutes, with a kind of explosive noise; and at last actual inflammation took place, with the decomposition and dissipation of the greatest part of the salt.*

That the increase of the conducting power of the water by the drop of saline solution had little or nothing to do with the effect, is evident from this circumstance. I introduced a quantity of strong lixivium of potash into the cones, and likewise concentrated sulphuric acid, separately, which are better conductors than solutions of the neutral salts; but there was very little sensible effect.

The same principles will apply to all the varieties of the electrical apparatus, whether containing double or single plates; and if the ideas developed in the preceding sections be correct, one property operating under different modifications is the universal cause of their activity.

* In this process ammonia was rapidly given off from the surface of the negative cone, and nitrous acid from that of the positive cone, and a white vapour was produced by their combination in the atmosphere above the apparatus.

X. *On some general Illustrations and Applications of the foregoing Facts and Principles, and Conclusion.*

The general ideas advanced in the preceding pages are evidently directly in contradiction to the opinion advanced by FABRONI, and which, in the early stage of the investigation, appeared extremely probable, namely, that chemical changes are the *primary* causes of the phænomena of GALVANISM.

Before the experiments of M. VOLTA on the electricity excited by the mere contact of metals were published, I had to a certain extent adopted this opinion; but the new facts immediately proved that another power must necessarily be concerned; for it was not possible to refer the electricity exhibited by the apposition of metallic surfaces to any chemical alterations, particularly as the effect is more distinct in a dry atmosphere, in which even the most oxidable metals do not change, than in a moist one, in which many metals undergo chemical alteration.

Other facts likewise soon occurred demonstrative of the same thing. In the VOLTAIC combination of diluted nitrous acid, zinc and copper, as is well known, the side of the zinc exposed to the acid is positive. But in combinations of zinc, water and diluted nitric acid, the surface exposed to the acid is negative; though if the *chemical* action of the acid on the zinc had been the cause of the effect, it ought to be the same in both cases.

In mere cases of chemical change likewise electricity is never exhibited. Iron burnt in oxygene gas, properly connected with a condensing electrometer, gives no charge to it during the process. Nitre and charcoal deflagrated in com-

munication with the same instrument do not by their agencies in the slightest degree affect the gold leaves. Solid pure potash and sulphuric acid made to combine in an insulated platina crucible produce no electrical appearances. A solid amalgam of bismuth and a solid amalgam of lead become fluid when mixed together: the experiment, I find, is connected with a diminution of temperature, but with no exhibition of electrical effects. A thin plate of zinc, after being placed upon a surface of mercury, and separated by an insulating body, is found positive, the mercury is negative: the effects are exalted by heating the metals; but let them be kept in contact sufficiently long to amalgamate, and the compound gives no signs of electricity. I could mention a great number of other instances of *pure chemical* action in which I have used all the means in my power to ascertain the fact, and the result has been constantly the same. In cases of effervescence, indeed, particularly when accompanied by much heat, the metallic vessels employed become negative, but this is a phænomenon connected with *evaporation*, the change of state of a body independent of chemical change, and is to be referred to a different law.*

* The change of the capacities of bodies in consequence of the alteration in their volumes, or states of existence by heat, is a continually operating source of electrical effects: and as I have hinted, page 36, it often interferes with the results of experiments on the electrical energies of bodies as exhibited by contact. It is likewise probably one of the sources of the capricious results of experiments of friction, in which the same body, according as its texture is altered, or its temperature changed, assumes different states with regard to another body. Friction may be considered as a succession of contacts, and the natural energies of bodies would probably be accurately exhibited by it, if the unequal excitation of heat or its unequal communication to the different surfaces did not interfere by altering unequally their electrical capacities. Of the elements of flint glass, silex is slightly negative with

I mentioned the glass plates of BECCARIA as affording a parallel to the case of combination in consequence of the different electrical states of bodies. In GUYTON DE MORVEAU'S experiments on cohesion, the different metals are said to have adhered to mercury with a force proportional to their chemical affinities. But the other metals have different electrical energies, or different degrees of the same electrical energy with regard to this body; and in all cases of contact of mercury with another metal, upon a large surface, they ought to adhere in consequence of the difference of their electrical states, and that with a force proportional to the exaltation of those states. Iron, which M. GUYTON found slightly adhesive, I find exhibits little positive electricity after being laid upon a surface of mercury, and then separated. Tin, zinc, and copper, which adhere much more strongly, communicate higher charges to the condensing electrometer: I have had no instrument sufficiently exact to measure the differences: but it would seem that the adhesion from the difference of electrical states must have operated in these experiments,* which being proportional to the electrical energies are, on the hypothesis before stated, proportional to the chemical affinities. How far cohesion in general may be influenced or occasioned by this effect of the difference of the electrical energies of bodies is a curious question for investigation.

Many applications of the general facts and principles to

regard to the metals, the soda is positive; and in contacts of glass with metals I find it exhibits the excess of the energy of the alkali: the case, as is well known, is the same in friction, the amalgam of the common machine is essential to its powerful excitation.

* Amalgamation undoubtedly must have interfered; but the *general* result seems to have been distinct.

the processes of chemistry, both in art and in nature, will readily suggest themselves to the philosophical enquirer.

They offer very easy methods of separating acid and alkaline matter, when they exist in combination, either together or separately, in minerals; and the electrical powers of decomposition may be easily employed in animal and vegetable analysis.

A piece of muscular fibre, of two inches long and half an inch in diameter, after being electrified by the power of 150 for five days, became perfectly dry and hard, and left on incineration no saline matter. Potash, soda, ammonia, lime, and oxide of iron were evolved from it on the negative side, and the three common mineral acids and the phosphoric acid, were given out on the positive side.

A laurel leaf treated in the same manner, appeared as if it had been exposed to a heat of 500° or 600° FAHRENHEIT, and was brown and parched. Green colouring matter, with resin, alkali, and lime, appeared in the negative vessel: and the positive vessel contained a clear fluid, which had the smell of peach blossoms; and which, when neutralized by potash, gave a blue-green precipitate to solution of sulphate of iron; so that it contained vegetable prussic acid.

A small plant of mint, in a state of healthy vegetation, was made the medium of connection in the battery, its extremities being in contact with pure water; the process was carried on for 10 minutes: potash and lime were found in the negatively electrified water, and acid matter in the positively electrified water, which occasioned a precipitate in solutions of muriate of barytes, nitrate of silver, and muriate of lime. This plant recovered after the process: but a similar one, that had been

electrified for four hours with like results, faded and died.* The facts shew that the electrical powers of decomposition act even upon living vegetable matter; and there are some phænomena which seem to prove that they operate likewise upon living animal systems. When the fingers, after having been carefully washed with pure water, are brought in contact with this fluid in the positive part of the circuit, acid matter is rapidly developed, having the characters of a mixture of muriatic, phosphoric, and sulphuric acids: and if a similar trial be made in the negative part, fixed alkaline matter is as quickly exhibited.

The acid and alkaline tastes produced upon the tongue, in GALVANIC experiments, seem to depend upon the decomposition of the saline matter contained in the living animal substance, and perhaps in the saliva.

As acid and alkaline substances are capable of being separated from their combinations in living systems by electrical powers, there is every reason to believe that by converse methods they may be likewise introduced into the animal œconomy, or made to pass through the animal organs: and the same thing may be supposed of metallic oxides; and these ideas ought to lead to some new investigations in medicine and physiology.

It is not improbable that the electrical decomposition of the neutral salts in different cases may admit of œconomical uses.

* Seeds, I find, when placed in pure water in the positive part of the circuit, germinate much more rapidly than under common circumstances; but in the negative part of the circuit they do not germinate at all. Without supposing any peculiar effects from the different electricities which however *may* operate, the phænomenon may be accounted for from the saturation of the water near the positive metallic surface with oxygene, and of that near the negative surface with hydrogene.

Well burned charcoal and plumbago, or charcoal and iron, might be made the exciting powers; and such an arrangement if erected upon an extensive scale, neutrosaline matter being employed in every series, would, there is every reason to believe, produce large quantities of acids and alkalies with very little trouble or expence.

Ammonia and acids capable of decomposition, undergo chemical change in the VOLTAIC circuit only when they are in very concentrated solution, and in other cases are merely carried to their particular points of rest. This fact may induce us to hope that the new mode of analysis may lead us to the discovery of the *true* elements of bodies, if the materials acted on be employed in a certain state of concentration, and the electricity be sufficiently exalted. For if chemical union be of the nature which I have ventured to suppose, however strong the natural electrical energies of the elements of bodies may be, yet there is every probability of a limit to their strength: whereas the powers of our artificial instruments seem capable of indefinite increase.

Alterations of electrical equilibrium are continually taking place in nature; and it is probable that this influence, in its faculties of decomposition and transference, considerably interferes with the chemical alterations occurring in different parts of our system.

The electrical appearances which precede earthquakes and volcanic eruptions, and which have been described by the greater number of observers of these awful events, admit of very easy explanation on the principles that have been stated.

Besides the cases of sudden and violent change, there must be constant and tranquil alterations in which electricity is

concerned, produced in various parts of the interior strata of our globe.

Where pyritous strata and strata of coal-blende occur, where the pure metals or the sulphurets are found in contact with each other, or any conducting substances, and where different strata contain different saline menstrua, electricity must be continually manifested; and it is very probable, that many mineral formations have been materially influenced, or even occasioned by its agencies.

In an experiment that I made of electrifying a mixed solution of muriates of iron, of copper, of tin, and of cobalt, in a positive vessel, distilled water being in a negative vessel, all the four oxides passed along the asbestus, and into the negative tube, and a yellow metallic crust formed on the wire, and the oxides arranged themselves in a mixed state round the base of it.

In another experiment, in which carbonate of copper was diffused through water in a state of minute division, and a negative wire placed in a small perforated cube of zeolite in the water, green crystals collected round the cube; the particles not being capable of penetrating it.

By a multiplication of such instances the electrical power of transference may be easily conceived to apply to the explanation of some of the principal and most mysterious facts in geology.

And by imagining a scale of feeble powers, it would be easy to account for the association of the insoluble metallic and earthy compounds, containing acids.

Natural electricity has hitherto been little investigated,

except in the case of its evident and powerful concentration in the atmosphere.

Its slow and silent operations in every part of the surface will probably be found more immediately and importantly connected with the order and œconomy of nature; and investigations on this subject can hardly fail to enlighten our philosophical systems of the earth; and may possibly place new powers within our reach.

EXPLANATION OF THE FIGURES.

Plate I.

Fig. 1, Represents the agate cups, mentioned page 4.

Fig. 2, Represents the gold cones, page 6.

Fig. 3, Represents the glass tubes, and their attached apparatus, page 21.

Fig. 4, Represents the two glass tubes, with the intermediate vessel, page 22.

In all the figures A B denote the wires, rendered one positively, the other negatively electrical; and C the connecting pieces of moistened amianthus.

Fig. 1.

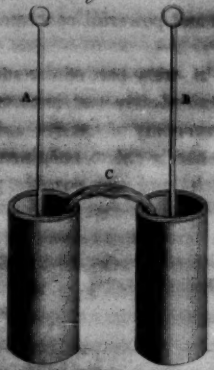


Fig. 2.

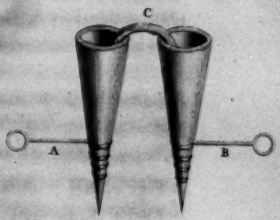


Fig. 3.

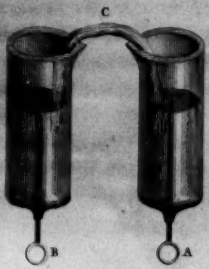
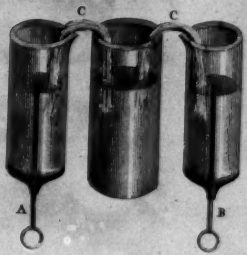


Fig. 4.



Basire sculp.

II. *On the Precession of the Equinoxes.* By the *Rev.* Abram Robertson, *M. A. F. R. S. Savilian Professor of Geometry in the University of Oxford.*

Read December 18, 1806.

PERHAPS the solution of no other problem, in natural philosophy, has so often baffled the attempts of mathematicians as that of determining the precession of the equinoxes, by the theory of gravity. The phenomenon itself was observed about one hundred and fifty years before the christian æra, but Sir ISAAC NEWTON was the first who endeavoured to estimate its magnitude by the true principles of motion, combined with the attractive influence of the sun and moon on the spheroidal figure of the earth. It has always been allowed, by those competent to judge, that his investigations relating to the subject evince the same transcendent abilities as are displayed in the other parts of his immortal work, THE MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY; but, for more than half a century past, it has been justly asserted that he made a mistake in his process, which rendered his conclusions erroneous.

Since the detection of this error, some of the most eminent mathematicians in Europe have attempted solutions of the problem. Their success has been various; but their investigations may be arranged under three general heads. Under the first of these may be placed such as lead to a wrong

conclusion, in consequence of a mistake committed in some part of the proceedings. The second head may be allotted to those in which the conclusions may be admitted as just, but rendered so by the counteraction of opposite errors. Such may be ranked under the third head as are conducted without error fatal to the conclusion, and in which the result is as near the truth as the subject seems to admit.

The authors of those investigations, of each of the three descriptions, are entitled to much praise. Their productions afford the most unquestionable proofs of great talents, great zeal, and great perseverance, exerted in the cultivation of science. The mistakes committed in those of the two first descriptions, and the obscurity and perplexity with which those of the third may be charged, are, in my opinion, to be attributed to the same cause, the uncultivated state of that particular department of the doctrine of motion, which constitutes the appropriate foundation for the solution of the problem. The department to which I allude is that of compound rotatory motion.

In consequence of this persuasion I have, in the first nine of the following articles, endeavoured to investigate the primary properties of compound rotatory motion from clear and unexceptionable principles. The disturbing solar force on the spheroidal figure of the earth is then calculated, and the angular velocity which it produces is afterwards compared with that of the diurnal revolution, by means of the properties of rotatory motion previously demonstrated. The quantity of annual precession is then calculated in the usual way, and also that of nutation, as far as they are produced by the disturbing force of the sun.

1. Let C, (Plate II. Fig. 1.) be the centre of two circular arcs AB, EF, which are the measures of the angles ACB, ECF; and let CB cut EF in D. Then, as the sectors ACB, ECD are similar, CA : CE :: AB : ED $= \frac{CE \times AB}{CA}$. But (Eu. 33. VI.) ED : EF :: the angle ACB : the angle ECF; and therefore $\frac{CE \times AB}{CA}$: EF :: < ACB ; < ECF. Consequently CE × AB × < ECF $=$ CA × EF × < ACB; and therefore AB : EF :: CA × < ACB : CE × < ECF.

2. Let ACB, GEF (Fig. 2.) be two angles, of which the arcs AB, GF are the measures; and the radii CA, EG not being necessarily equal, let the sines BK, FQ be equal to one another. Let BH, FM be tangents to the curves; and let HD, MN be parallel to CA, EG respectively, and meet BK, FQ in D, N, as represented. Then as CBH, BDH are right angles, the triangles CBK, BHD are equiangular, and CB : BK :: BH : HD or its equal KL, if HL be drawn parallel to BK, and meet CA in L. Consequently $BK = \frac{CB \times KL}{BH}$. For the same reasons, if MP be parallel to FQ and meet EG in P, $FQ = \frac{EF \times QP}{FM}$; and therefore as BK, FQ are, by hypothesis, equal, $\frac{CB \times KL}{BH} = \frac{EF \times QP}{FM}$. Hence CB × KL × FM $=$ EF × QP × BH, and BH : FM :: CB × KL : EF × QP.

3. If, therefore, we suppose straight lines CH, EM to be drawn, and that the angles BCH, FEM are indefinitely small, and generated in the same time by the revolution of CB, EF respectively, then BH, FM may be considered as circular arcs, and by article 1, BH : FM :: CB × < BCH : EF × < FEM. Hence by article 2, (and 11. V.) CB × < BCH : EF × < FEM :: CB × KL : EF × QP, and therefore < BCH : < FEM :: KL

: QP. Consequently, during the generation of the angles BCH, FEM, the angular velocity of CB is to the angular velocity of EF as KL to QP.

4. If ACB (Fig. 3.) be any plane angle, and from any point E, EF be drawn perpendicular to AC and EH perpendicular to CB, then the angle FEH is equal to the angle ACB. For let HK be drawn perpendicular to AC, and let EH be produced to G. First let ACB be an obtuse angle, and then $\angle ACB = \angle KHC + \angle HKC + \angle KHC + \angle CHG = \angle KHG$ = (29. I.) $\angle FEH$. Secondly, ACB being an acute angle, the right angle $GHC = \angle GHK + \angle KHC = \angle KHC + \angle HCK$ = (15. I.) $\angle KHC + \angle ACB$. Consequently the angle $ACB = \angle GHK = \angle FEH$.

5. Let G (Fig. 4.) be the centre of gravity of a body, and AB, DC two axes passing through G; and, while the body revolves round AB, let AB and consequently the whole body revolve round DC, the periodical times of these revolutions not being necessarily equal; it is required to determine the direction and angular velocity with which any particle of the body revolves in consequence of this compound motion.

Suppose the simple motion of the body about AB to be such, that during the revolution the parts towards D from AB would rise above the plane, on which the figure is drawn, and the parts towards C sink below it. And suppose the simple motion about DC to be such, that during the revolution the parts towards A from DC would rise above the plane of the figure, and those towards B sink below it. Let P (Fig. 5.) be a particle of the body, above the plane, and let PR be a perpendicular to the said plane.* With the centre G suppose

* The axis DC, and the line RM are intentionally omitted in Fig. 5, with a view to prevent confusion in the figure.

a spherical superficies to be described, passing through P, and let ADBC be the great circle of this sphere in the plane on which the whole is represented. Let the straight lines EF, HK pass through R, and be perpendicular to AB, DC respectively; and let EPF, HPK be lesser circles of the sphere, EF being a diameter of the one, and HK a diameter of the other. Then it is evident, that by the simple motion of the body about AB only the particle P would move in the circumference FPE; and by the simple motion of the body about DC only, P would move in the circumference HPK. Let the indefinitely small arcs P*s*, P*q* be those which P would describe in equal times with the revolutions about AB, DC separately, and let the parallelogram P*qps* be completed on the spherical superficies. Then it is evident, from the composition of motion, that the direction and velocity of P, in consequence of the compound motion, is as the diagonal P*p* of the parallelogram P*qps*.

Let R*r*MN be the orthographical projection of P*qps* on the plane ADBC, and then as PR is perpendicular to this plane, it is evident that R*r*MN is a parallelogram, and that its diagonal RM is the direction and velocity of P in the projection, in consequence of the compound motion. It therefore follows, from article 3, as RN is the angular velocity about the axis AB, and R*r* that about the axis DC, that RM is the angular velocity about the axis, round which the body is caused to revolve by the compound motion.

6. The same things being supposed, and the parallelogram R*r*MN being the same in Fig. 4, as in Fig. 5, let RM produced meet the circumference in L and Q, and the diameter TGS, at right angles to LQ, is the axis sought.

The same axis may be obtained in the following manner.

In GB take GV equal to RN; and in GC take GW equal to Rr, and VW being drawn it will be parallel to the axis TS. For as NR is perpendicular to AB, and Rr or NM to DC, by article 4, the angle RNM, or (34. I.) its equal $R\nu M$, is equal to the angle VGW. Also, on account of the equals, VG : GW :: Mr : νR, and therefore (6. VI.) the angles νRM, GWV are equal. Let TS meet HK in O, and LQ in I; and let DC meet HK in X. Then as the angle OIR is equal to the angle OXG, each being a right one, and as the angles IOR, XOG are equal, the angle IRO, or $MR\nu$ is equal to the angle OGX. Consequently the alternate angles XGO, GWV are equal, and therefore TS, VW are parallel. Hence it is evident that if the axes AB, DC, and also GV, GW the angular velocities round them be given, the axis TS is easily found, being parallel to VW. It is proper to observe that GV, GW are to be set off on that side of TS towards which the body is moving, in consequence of the revolutions round DC, AB.

7. From the last article it is evident that VW is equal to RM, and consequently equal to the angular velocity, with which the body revolves about the axis TS. If therefore CGB be a right angle, then the angular velocity $VW = \sqrt{VG^2 + GW^2}$. In other cases the value of VW may be easily calculated by plane Trigonometry.

8. It is to be remarked, for the sake of precision, that the linear velocity, of any point, is as the angular velocity multiplied into the radius of the circle in whose circumference it revolves. Thus the linear velocity Ps (Fig. 5.) of the point P in the circumference FPE, is as its angular velocity in the same, multiplied into the radius of the circle FPE, as is evi-

dent from article 3. In the following articles, linear velocity is meant when no adjective is annexed to the word velocity.

9. Let AB, DC (Fig. 6.) be the two axes about which separately the body would revolve, as stated in article 5, and let TS be the axis about which it revolves, in consequence of a combination of these two revolutions. Let TE be at right angles to AB, and meet it in H, and let TF be at right angles to DC and meet it in K; and let GV, GW be the angular velocities about AB, DC, as in the preceding articles. Then it follows, from the last article, that the velocity of the point T, by the revolution about the axis AB only, is equal to GV×HT. And as this velocity is in the direction of a tangent at T to the circle of which TE is a diameter, and as this circle is perpendicular to the plane ADBC, the direction of this velocity is evidently perpendicular to the plane ADBC. The direction of this velocity of the point T is also upwards from the plane of the figure, agreeable to the statement in article 5. Again, by the revolution about the axis DC only, the velocity of T is equal to GW×KT, and, for the foregoing reasons, the direction of this velocity of the point T is perpendicularly downwards below the plane, according to article 5. Now as TS is the axis about which the body revolves, in consequence of the combined revolutions about AB, DC, every point in TS is rendered quiescent by the compound motions. It is therefore evident that GV×HT=GW×KT.

10. The revolutions about DC, AB may be supposed to be caused by instantaneous impulses at A and D, made at the same time, or at different times; or they may be supposed to be occasioned by the agency of constant forces, like that of

gravity. For if the causes be adequate to the production of the same velocities, taken separately, and in the same directions, the velocity and direction of a particle will be the same from their combined influence upon it, whether these causes be impulses or constant forces.

As the body is understood to be in free space, if the causes of the revolutions, taken separately, be instantaneous impulses, and made at the same time, immediately after their agency the body will revolve about the axis TS, and it will continue so to revolve with an uniform velocity. If whilst the body is revolving with an uniform velocity about the axis DC, a constant force begin to act at D, so as to cause a tendency to revolution about AB, as stated in article 5, and continue afterwards to act at T, the pole of the new axis, from a combination of the constant agency at the new pole and the uniform velocity about DC, the axis TS will incessantly shift its position.

Such exactly are the circumstances to which the earth is subject as to the production of the precession of the equinoxes. At the vernal equinox, for instance, a straight line drawn from the centre of the sun to that of the earth is in the plane of the equator, and therefore, as equal portions of the protuberant matter of the earth are above and below the ecliptic, the attractive power of the sun has no tendency to alter the position of the equator. But, in consequence of the earth's motion in its orbit, it very soon after the equinox presents a different position of the equator to the sun. The equilibrium of the protuberant parts of the earth, above and below the ecliptic, and towards the sun, is then done away,

and the attraction of the sun on that side, where the greatest quantity of protuberant matter is, tends to bring down the equator into the ecliptic, or to cause the earth to revolve about a diameter of the equator. This attractive influence of the sun gradually increases a little till the summer solstice; it then gradually decreases in the same degree till the autumnal equinox, when it vanishes. From the autumnal equinox to the winter solstice it again gradually increases a little; and it then gradually decreases in the same degree till the vernal equinox, when it again vanishes. This recurrence and continuance of action is annually repeated.

Similar observations apply to the attraction of the moon on the protuberant parts of the earth. When a straight line drawn from her centre to that of the earth is in the plane of the equator, the attractive influence of the moon has no tendency to change the position of the equator, but in other situations, the attraction of the moon tends to bring the equator of the earth into the plane of the moon's orbit, or causes the earth to move round a diameter of the equator. The recurrences of the moon's action on the protuberant parts of the earth, and the times of their continuing, are repeated every month.

These effects of the sun and moon are to be considered separately; and for the reasons already stated, each of the actions, combined with the diurnal revolution of the earth, may be considered as a particular case of compound rotatory motion. It is needless, however, after investigating the effects of the sun's action, and expressing them in general formulæ, to go over the same steps for ascertaining those of the moon,

as they may be inferred from the former, after making due allowance for the different circumstances under which these two remote bodies act on the protuberant parts of the earth.

I now proceed to estimate the force with which the sun tends to cause the earth to revolve about a diameter of the equator.

11. Let S be the centre of the sun (Fig. 7.) C that of the earth; P, L the poles, PL the axis; and let a plane passing through SC, PL cut the earth in the meridian PEAQB. Let EQ be the diameter of the equator, and let DF, the diameter of the spheroid in the plane SPCL, be at right angles to SC. Let SC cut the meridian EPQL in A, B; and G being supposed a particle of matter in this meridian, let GH parallel to SC meet DF in H, and let SG be drawn. Let M be the quantity of matter in the sun, or its absolute attracting power, and then $\frac{M}{SG^2}$ is its force upon the particle G, in the direction SG, and $\frac{M}{SC^2}$ is its force upon a particle at C, in the direction SC. But a force whose power and direction is as GS is equal to a force whose power and direction is as GC, together with a force whose power and direction is as CS; and as the force whose power and direction is as GC, is directed to the centre it has no tendency to alter the position of the axis PL, and therefore may be neglected in the present enquiry. Now, by Mechanics, $SG : SC :: \frac{M}{SG^2} : \frac{M \times SC}{SG^3} =$ the force of the sun on the particle G, in the direction CS or HG. Now as the distance of the sun from the earth is indefinitely great when compared to the diameter DF, its force on any particle in DF is equal to its force on a particle at C, and therefore the sun's force on a particle at H is as $\frac{M}{SC^2}$. Consequently, as the sun's

force on the particle G, in the same direction, is as $\frac{M \times SC}{SC^3}$, the disturbing force of the sun, by its action on the particle G is $\frac{M \times SC}{SG^3} - \frac{M}{SC^2}$, or $\frac{M \times SC}{\overline{SC - GH}|^3} - \frac{M}{SC^2}$, for SG may be considered as equal to SC — GH. But as SC is indefinitely great with respect to GH, $\frac{SC}{\overline{SC - GH}|^3}$, by actual division may be considered as equal to $\frac{1}{SC^2} + \frac{3GH}{SC^3}$, and therefore the disturbing force on the particle at G is $\frac{3M \times GH}{SC^3}$.

Let K be a particle in the meridian, but on the opposite side of DF to that on which G is situated. Let KN, parallel to SC, meet DF in N; and suppose SK to be drawn. Then the force of the sun on K being $\frac{M}{SK^2}$, for the same reasons as before, its force upon it in the direction of SC or KN is $\frac{M \times SC}{\overline{SC + KN}|^3}$, and after a reduction similar to the foregoing, the sun's disturbing force on K is $-\frac{3M \times KN}{SC^3}$.

Hence it is evident, supposing M and SC to be constant, that the disturbing force of the sun on any particle in the meridian PELQ is as the distance of the particle from DF; and that the sign of the force in the half DAF nearest to S is positive, but the sign of the force in the other half DBF is negative. This difference of the signs indicates that the particles on the opposite sides of DF have a directly opposite tendency, as to direction, in affecting the position of the axis PL, or equator EQ; and the same is evident from the following considerations. As the disturbing force is as its distance from DF, it has no effect on particles in DF, and therefore the inertia* of

* By this expression that part of the inertia is meant which opposes the disturbing force of the sun; and the same is to be understood in the following expressions.

particles in DF may be considered as equal to the sun's disturbing force, on the principle of action and reaction being equal as to magnitude, but directly contrary as to direction. But on the side of DF nearest to S the disturbing force is greater than the inertia of any particle G, and it therefore urges the particle from DF towards S, by a pressure whose direction and power is as HG. On the side of DF opposite to S the disturbing force is less than the inertia of any particle K, and therefore the inertia of K opposes the disturbing force of the sun by a pressure whose direction is from N towards K, and whose power is as NK.

12. As, by the nature of the spheroid, PELQ is an ellipse, let GK be the diameter conjugate to DF, and let VI, parallel to DF, meet it in T, and AB in R; and then VI is bisected in T. Let RI be bisected in v, and let w, q be two points in RI, equally distant from R and I respectively. Let $a =$ EC, and $d =$ the disturbing force of the sun at the distance of EC from DF. Then by the preceding article, $a : RC :: d : \frac{d}{a} \times RC$ = the force at R, or at any point in VI, as any two points in VI are equally distant from DF. Now it is evident that the disturbing force on a particle at R, or on any particle in AC, has no power to turn the ellipse about C; but the force on a particle at w tends to turn the ellipse about the centre, for it is applied at the end of the lever Rw. Consequently, by what has been already proved in this article, and by the property of the lever, the force on w to turn the ellipse is $\frac{d}{a} \times RC \times Rw$. For the same reasons, the force on q to turn the ellipse is $\frac{d}{a} \times RC \times Rq$, and therefore the force on w and q combined, to turn the ellipse, is $\frac{d}{a} \times RC \times RI$, for $Rw + Rq = RI$.

Hence, as Rv or $\frac{RI}{2}$ expresses the number of particles in Rv or vI, it follows that the force of all the particles in RI, to turn the ellipse, is $\frac{d}{a} \times RC \times RI \times \frac{RI}{2} = \frac{d}{2a} \times RC \times RI^2$. In the same way it may be proved that the force of all the particles in VR, to turn the ellipse, is $\frac{d}{2a} \times RC \times RV^2$. But the force of the particles in RI tends to turn the ellipse upwards in the direction FAD, and the force of the particles in RV tends to turn it downwards in the contrary direction DAF. The force of all the particles in VI, therefore, to turn the ellipse, is $\frac{d}{2a} \times RC \times \overline{RV^2 - RI^2}$. But as TV is half the sum of RV, RI, it follows that RT is half their difference, and therefore $RV^2 - RI^2 = VI \times 2RT$. Consequently the force of the particles in VI, to turn the ellipse, is $\frac{d}{2a} \times RC \times VI \times 2RT = \frac{d}{a} \times RC \times VI \times RT$.

13. Let $c = CG$, $b = CD$, $f = GH$, $g = CH$, $y = CR$, and $x = CT$. Then by similar triangles, $c : f :: x : y = \frac{fx}{c}$; and $c : g :: x : \frac{gx}{c} = RT$. Also, by the property of the ellipse, $c^2 : b^2 :: GT \times TK : TV^2 :: \overline{c+x} \times \overline{c-x} : TV^2 :: c^2 - x^2 : TV^2$. Consequently $TV = \frac{b}{c}\sqrt{c^2 - x^2}$, and $VI = \frac{2b}{c}\sqrt{c^2 - x^2}$, and the force of the particles in VI, to turn the ellipse, is $\frac{d}{a} \times y \times \frac{2b}{c}\sqrt{c^2 - x^2} \times \frac{gx}{c} = \frac{2bdfgx}{ac^2}\sqrt{c^2 - x^2}$, by putting $\frac{fx}{c}$ for y. The fluxion of this force is therefore $\frac{2bdfgx^2}{ac^2}\sqrt{c^2 - x^2} \times \dot{y} = \frac{2bdfgx^2}{ac^2}\sqrt{c^2 - x^2} \times \frac{f\dot{x}}{c} = \frac{2bdfgx^2\dot{x}}{ac^3}\sqrt{c^2 - x^2}$, as $\frac{f\dot{x}}{c} = \dot{y}$. The fluent of this expression may be found in the following manner.

14. The fluxion of $x \times \overline{c^2 - x^2}^{\frac{1}{2}}$ is $\dot{x} \times \overline{c^2 - x^2}^{\frac{1}{2}} + \frac{1}{2} \times x \times \overline{c^2 - x^2}^{-\frac{1}{2}} \times -2x\dot{x} = \dot{x} \times \overline{c^2 - x^2}^{\frac{1}{2}} - x^2\dot{x} \times \overline{c^2 - x^2}^{-\frac{1}{2}} = \dot{x}\sqrt{c^2 - x^2} - x^2\dot{x} \times \overline{c^2 - x^2}^{-\frac{1}{2}} = \dot{x} \times \overline{c^2 - x^2}$

$\times\sqrt{c^2-x^2} = 3\sqrt{c^2-x^2}\times x^2\dot{x} = c^2\dot{x}-x^2\dot{x}\times\sqrt{c^2-x^2} =$
$3\sqrt{c^2-x^2}\times x^2\dot{x} = c^2\dot{x}\sqrt{c^2-x^2}-x^3\dot{x}\sqrt{c^2-x^2}$. Consequently the fluxion of $\frac{x\times c^2-x^2|^{\frac{1}{2}}}{4}$ is $\dot{x}\sqrt{c^2-x^2}\times\frac{c^2}{4}-x^2\dot{x}\sqrt{c^2-x^2}$; and the fluxion of $\frac{2bf}{c^2}\times\frac{x\times c^2-x^2|^{\frac{1}{2}}}{4}$ is $\frac{2bf}{c^2}\times\dot{x}\sqrt{c^2-x^2}\times\frac{c^2}{4}-\frac{2bf}{c^2}\times x^2\dot{x}\sqrt{c^2-x^2}$. Hence the fluent of $\frac{2bf}{c^2}\times\dot{x}\sqrt{c^2-x^2}\times\frac{c^2}{4}+\frac{2bf}{c^2}\times x^2\dot{x}\sqrt{c^2-x^2}$ = the fluent of $\frac{2bf}{c^2}\times\dot{x}\sqrt{c^2-x^2}\times\frac{c^2}{4}$ — the fluent of $\frac{2bf}{c^2}\times x^2\dot{x}\sqrt{c^2-x^2} = \frac{2bf}{c^2}\times\frac{x\times c^2-x^2|^{\frac{1}{2}}}{4}$; and therefore by transposition, the fluent of $\frac{2bf}{c^2}\times\dot{x}\sqrt{c^2-x^2}\times\frac{c^2}{4}-\frac{2bf}{c^2}\times\frac{x\times c^2-x^2|^{\frac{1}{2}}}{4}$ = the fluent of $\frac{2bf}{c^2}\times x^2\dot{x}\sqrt{c^2-x^2}$. Again, as VI is equal to $\frac{2b}{c}\sqrt{c-x}$, the fluxion of the area DVIF is $\frac{2b}{c}\sqrt{c-x}\times\dot{y} = \frac{2bf}{c^2}\times\dot{x}\sqrt{c^2-x^2}$, and therefore the fluent of $\frac{2bf}{c^2}\times\dot{x}\sqrt{c^2-x^2}$ is the area DVIF; and the fluent of $\frac{2bf}{c^2}\times\dot{x}\sqrt{c^2-x^2}\times\frac{c^2}{4}$ is the area DVIF $\times\frac{c^2}{4}$. Consequently the area DVIF $\times\frac{c^2}{4}-\frac{2bf}{c^2}\times\frac{x\times c^2-x^2|^{\frac{1}{2}}}{4}$ = the fluent of $\frac{2bf}{c^2}\times x^2\dot{x}\sqrt{c^2-x^2}$, and therefore $\frac{df_g}{ac^2}\times$ area DVIF $\times\frac{c^2}{4}-\frac{df_g}{ac^2}\times\frac{2bf}{c^2}\times\frac{x\times c^2-x^2|^{\frac{1}{2}}}{4}$ = the fluent of $\frac{df_g}{ac^2}\times\frac{2bf}{c^2}\times x^2\dot{x}\sqrt{c^2-x^2}$, or the force of the particles in the area DVIF to turn the ellipse. Hence, when x becomes equal to c, the force of all the particles in the semi-ellipse DGF, to turn the ellipse, is equal to the semi-ellipse DGF $\times\frac{df_g}{ac^2}\times\frac{c^2}{4}$ = the semi-ellipse DGF $\times\frac{d}{a}\times\frac{f}{4}\times g$. By article 11, therefore, the force of all the particles in the whole ellipse, which tend to turn it about the centre, is the whole ellipse PELQ $\times\frac{d}{a}\times\frac{f}{4}\times g$.

15. The other circumstances as to the figure being the same as before, let the straight line GV (Fig. 8.) touch the ellipse in G, meet QE in V and CM parallel to GH in M. Let GI be perpendicular to QE and meet it in I, and let GH meet EQ in T. Then, by a well known property of the ellipse, CI : IT :: EQ : its parameter; and therefore by the nature of a parmeter, or three proportionals, CI : IT :: CE^2 : CP^2, and $IT = \frac{CI \times CP^2}{CE^2}$. Hence $CT = CI - IT = CI - \frac{CI \times CP^2}{CE^2}$; and by another well known property of the ellipse, $CV = \frac{CE^2}{CI}$. Consequently $CT \times CV = CE^2 - CP^2$. Now as MAC is part of the straight line drawn from the centre of the sun to that of the earth, the angle ACE is the sun's declination, and as DC is perpendicular to AC, the angle ECD is the complement of the declination. Let $m =$ the sine of the declination, and $n =$ its cosine, and then, radius being 1, CT : CH :: $1 : m$, and CV : CM or (34. I.) its equal GH :: $1 : n$. By multiplication therefore, $CT \times CV : CH \times GH :: 1 : mn$. Consequenty, using the same notation as in the last article, and putting $e = CP$, as $CT \times CV = CE^2 - CP^2$, we have $a^2 - e^2 : fg :: 1 : mn$, and $fg = \overline{a^2 - e^2} \times mn$. If therefore X denote the area of the ellipse PELQ, by the last article, the force of all the particles to turn the ellipse is $= \frac{d}{a} \times \frac{mn}{4} \times \overline{a^2 - e^2} \times X$.

16. Let PELQ be the same ellipse in Fig. 9, as in the 7th and 8th figures. Let the spheroid be cut by a plane parallel to PELQ, (Fig. 9.) and let the section be the ellipse HKNG; and let this ellipse be supposed to be above the plane of the paper on which PELQ is represented. Again, let the spheroid be cut by a plane passing through PL, and perpendicular to the plane PELQ, and let the line of common section of this

plane with the ellipse HKNG be HN, and let this ellipse be cut by the plane of the equator in the line KG. Then HN (14. XI.) is parallel to PL, and KG to EQ; and therefore (10. XI.) HN, KG cut one another at right angles. Again, as the axis PL is perpendicular to the plane of the equator, the plane of the equator is perpendicular (18. XI.) to the ellipse PELQ. The planes passing through the centre of the spheroid and the lines HN, KG are therefore perpendicular to the ellipse PELQ, and consequently (19. XI.) the straight line passing through the centre of the spheroid, and the point in which HN, KG cut one another, is perpendicular to HN, KG and also to PL, EQ. Consequently as the equator is a circle, KG (8. III.) is bisected by HN; and as HN is parallel to PL, it is a double ordinate to the diameter of the equator passing through the point in which HN, KG cut one another, and is therefore bisected in this point. Hence, as by a well known property of the spheroid, the ellipses PELQ, HKNG are similar, it is evident that KG is the transverse, and HN the conjugate axis of the ellipse HKNG.

Let $u =$ the distance of the centre of the ellipse HKNG from the centre of the speroid, and then as the points E, K, G, Q are in the circumference of the equator, a straight line drawn from the centre of the spheroid to K or G is equal to a, and half of the straight line $KG = \sqrt{a^2 - u^2}$. Again, as the ellipses PELQ, HKNG are similar, $a : e :: \sqrt{a^2 - u^2} : \frac{e}{a}\sqrt{a^2 - u^2} =$ half of HN; and $a^2 : a^2 - u^2 :: X : \frac{a^2 - u^2}{a^2} \times X =$ the area of the ellipse HKNG. Hence, in order to find the disturbing force of the sun on the ellipse HKNG, instead of a^2 in the expression $\frac{d}{a} \times \frac{m}{4} \times \overline{a^2 - e^2} \times X$ we are to put $a^2 - u^2$, instead of

e^2 we are to put $\frac{e^2}{a^2} \times \overline{a^2 - u^2}$, and instead of X we are to put $\frac{a^2 - u^2}{a^2}$ X. These substitutions being made, we have $\frac{d}{a} \times \frac{mn}{4} \times \frac{a^2 - e^2}{a^2} \times \overline{a^2 - u^2} \times \frac{a^2 - u^2}{a^2}$ X for the sun's disturbing force on the ellipse HKNG, tending to turn the ellipse about an axis passing through C and perpendicular to the plane PELQ. This expression for the force being multiplied by $\dot{u}$, gives the fluxion of the force or that part of the spheroid between the ellipses PELQ, HKNG; and the fluent of this, when u becomes equal to a is $\frac{d}{a} \times \frac{mn}{4} \times \frac{a^2 - e^2}{a^2} \times \frac{8a^3}{15}$ X. Consequently the double of this, *viz.* $\frac{d}{a} \times mn \times \overline{a^2 - e^2} \times \frac{4a}{15}$ X expresses the sun's disturbing force on the whole spheroid. Hence if $Z = \frac{4a}{3}$ X, which expresses the solid content of the spheroid, the force on the whole spheroid is $\frac{d}{a} \times \frac{mn}{5} \times \overline{a^2 - e^2} \times Z$. Let this be called the librating force or pressure, or the force causing libration.

17. It is evident, from the manner in which the librating pressure is calculated, that the whole of the disturbing force is occasioned by the protuberance of the spheroid above the greatest inscribed sphere. For if PELQ were a sphere, as VI (Fig. 7.) is parallel to the diameter DF, and AC perpendicular to it, the straight line VI (3. III.) would be bisected in R; and therefore the disturbing forces, above and below AC would exactly counteract one another.

Let DCF (Fig. 9.) denote a plane perpendicular to the straight line SC, then it is evident that the librating pressure tends to move the earth about that diameter of the equator, which is the common section of the equator and the plane DCF. For the sake of precision hereafter let this diameter of the equator be called the axis of libration.

The point E of the equator, nearest the sun, is at the distance of a quadrant from either extremity of the axis of libration. For, by hypothesis, SC is at right angles to the plane DCF, and therefore the axis of libration, which is in this plane, is at right angles to SC. Again, as PC the axis of the earth is at right angles to the plane of the equator, the axis of libration, which is also in the equator, is at right angles to CP. The axis of libration, therefore, being at right angles to CS, CP in the plane PELQ, is at right angles to CE in the same plane.

18. Let ADBE (Fig. 10.) be an oblate spheroid of which AB is the transverse axis, DE the conjugate axis, and C the centre. Let AKMBLH be the equator of this spheroid, and consequently at right angles to ADBE the generating ellipse. Let the spheroid be cut through DCE by a plane DMEL at right angles to ADBE, and let MCL be its common section with the equator. Then (19. XI.) MCL is at right angles to ADBE, and therefore ACM is a right angle; and as ACD is a right angle, AC is at right angles to the plane DMEL, and consequently at right angles to any plane parallel to it. Let the spheroid be cut by a plane parallel to DMEL, and let the common section of this plane with the spheroid be the ellipse FKGH. Let this ellipse cut the plane ADBE in the straight line F*r*G, and the plane of the equator in K*r*H, the point *r* being the centre of this last formed ellipse, or that point in which it meets AB. Then, by a well known property of the spheroid, the ellipses DMEL, FKGH are similar, and the area of the first is to that of the other as CA^2 or its equal CM^2 to rK^2. Put $a = AC$ or CM, $e = DC$, $x = Cr$, and the force of each particle in the spheroid being as its distance from the

plane DMEL, let v be the force of a particle at A. Let $p =$ the area of a circle whose diameter is 1. Then $4pae =$ the area of the ellipse DMEL, and as $\overline{a+x} \times \overline{a-x} = a^2-x^2 =$ rK^2, $a^2 : a^2-x^2 :: 4pae : \frac{4pe}{a} \times \overline{a^2-x^2} =$ the area of the ellipse FKGH. Again, $a : x :: v : \frac{vx}{a} =$ the force of a particle at r, and therefore $\frac{4pev}{a^2} \times \overline{a^2 x-x^3} =$ the force of all the particles in the ellipse FKGH. Now as this force acts at r, by the property of the lever, the power of the ellipse FKGH, to turn the spheroid, either about DE or ML as an axis, is $\frac{4pev}{a^2} \times$ $\overline{a^2 x^2-x^4}$; and the fluxion of this force is $\frac{4pev}{a^2} \times \overline{a^2 x^2 \dot{x}-x^3 \dot{x}}$. The fluent of this, when x becomes equal to a, is $\frac{8pa^3 ev}{15}$; and the double of this, for the force of the whole spheriod, is $\frac{16pa^3 ev}{15}$. Hence it is evident that if the force of each particle in the spheroid, to cause a revolution, be as its distance from the plane DMEL, the particles on one side of this plane having a tendency to cause a revolution in one direction, and the particles on the other side of the plane having an equal tendency to cause a revolution in the same direction, then the pressure with which the spheroid is urged to revolve, either about DE or ML, is as $\frac{16pa^3 ev}{15}$. This force is equal to $\frac{av}{5}$ Z, if Z be put equal to $\frac{16pa^3 e}{3}$, the solid content of the spheroid.

19. As the librating force $\frac{d}{a} \times \frac{mn}{5} \times \overline{a^2-e^2} \times$ Z, ascertained in article 16, and the force $\frac{av}{5}$ Z, obtained in the last article, are calculated on the same hypothesis, *viz.* that the force of a particle is as its distance from the plane DCF in Fig. 9, or the plane DMEL in Fig. 10, if they produce equal angular velocities, the spheroids in the two figures being equal in every

respect, and all other circumstances being the same, the forces themselves must be equal. Now at either of the equinoxes the other circumstances are exactly the same in the two figures. At the vernal equinox, for instance, the straight line SACB in Fig. 9. must be in the plane of the equator, and therefore the plane DCF, perpendicular to AB, at this time must pass through the poles P, L. At the equinox, therefore, the straight line SACB, and the plane DCF in Fig. 9. are justly represented by ACB, and DMEL in Fig. 10. Hence the librating force $\frac{d}{a} \times \frac{mn}{5} \times \overline{a^2 - e^2} \times Z$ at the commencement of its action, at the equinox, applies to Fig. 10, and at its commencement it is equally efficacious to cause revolution about DE or ML. We are therefore enabled to compare the effect of the librating force, or the revolution it is capable of producing, at the equinox, about ML, with the diurnal revolution of the earth about DE, in the following manner.

It being admitted that each of the two forces, stated in the beginning of the article, produces the same angular velocity, then $\frac{d}{a} \times \frac{mn}{5} \times \overline{a^2 - e^2} \times Z = \frac{av}{5} Z$, and therefore $d \times mn \times \frac{a^2 - e^2}{a^2} = v$. But if a constant force act for a given time t, and cause the body to move on which it acts, the velocity generated from the commencement of the motion is as the force. Consequently, as v denotes the force acting on a particle at A, during the given time t, and as the forces acting on the other particles of the spheroid are proportional to their distances from the plane DMEL; the angular velocity of A, acquired in the given time t, is also accurately expressed by v. If therefore the force $\frac{av}{5} Z$ cease to act at the end of the given time t, the point A, as the spheroid is in free space, will afterwards revolve

with the uniform angular velocity v; but by the doctrine of constant forces, the angle described by A, during the action of $\frac{av}{5}$ Z is equal to $\frac{v}{2} = \frac{d}{2} \times mn \times \frac{a^2 - c^2}{a^2}$.

20. Let AB (Fig. 6.) represent that diameter of the equator about which the librating force begins to cause revolution at the equinox. Let G be the centre of the earth, and in GB let GV be taken equal to $\frac{v}{2}$, or $\frac{d}{2} \times mn \times \frac{a^2 - c^2}{a^2}$. Let w denote the angular diurnal velocity of the earth about its axis DC: and in GC let GW be taken equal to w. The points V, W being joined, let TGS be drawn parallel to VW, and by article 6, TS is the axis about which the earth will now revolve, in consequence of the diurnal revolution being combined with the libration about AB. From T, a pole of this axis, let TK be drawn perpendicular to DG. Then, by article 9, GW : GV :: GK : KT. But as the angle DGT is extremely small, GK may be considered as equal to the radius, and the arc DT as equal to its sine. Consequently, using the notation already specified, and considering a as radius, $w : \frac{d}{2} \times mn \times \frac{a^2 - c^2}{a^2} :: a : \frac{ad}{2w} \times mn \times \frac{a^2 - c^2}{a^2}$ = the angular velocity caused by the librating force. Our next object is to find the value of d in known terms.

21. If t be put for the time of the earth's diurnal revolution round its axis, and T be put for the earth's annual revolution round the sun, then $\frac{w^2}{a}$ is equal to the centripetal force on a body revolving at the equator in the time t, with the velocity w; and, using the notation of article 11, $\frac{M}{SC^2}$ is equal to the centripetal force of the sun on the earth. By the doctrine of centripetal forces therefore $\frac{M}{SC^2} : \frac{w^2}{a} :: \frac{SC}{T^2} : \frac{a}{t^2}$, and $\frac{a \times M}{SC^2 \times t^2} =$

$\frac{w^2 \times SC}{a \times T^2}$, and $M = \frac{w^2 \times SC^2 \times t^2}{a^3 \times T^2}$. But, by article 11, the disturbing force of the sun on a particle at G (Fig. 7.) is equal to $\frac{3M \times GH}{SC^3}$; and at the distance a from DF the disturbing force is $\frac{3M \times a}{SC^3}$. The foregoing value of M being substituted for it in this expression, we have $\frac{3w^2 \times t^2}{a \times T^2} = d$, the sun's disturbing force at the distance a from DF.

This value of d being put for it in the expression at the end of the last article, it follows that the angular velocity of libration, at its commencement at the equinox, is to the uniform angular diurnal velocity as $\frac{3w \times t^2}{2T^2} \times mn \times \frac{a^2 - c^2}{a^2}$ to w, or as $\frac{3t^2}{2T^2} \times mn \times \frac{a^2 - c^2}{a^2}$ to 1. But, according to the preceding notation, $t : i :: 360^\circ : \frac{360i}{t}$ = the uniform angular diurnal velocity, and therefore $1 : \frac{3t^2}{2T^2} \times mn \times \frac{a^2 - c^2}{a^2} :: \frac{360i}{t} : 360 \times \frac{3ti}{2T^2} \times mn \times \frac{a^2 - c^2}{a^2}$ = the angular velocity of libration, at its commencement at the equinox. But as the product mn is the only variable quantity which enters into the value of the librating force, obtained in article 16, it is evident that $360 \times \frac{3ti}{2T^2} \times mn \times \frac{a^2 - c^2}{a^2}$ expresses the momentary angular velocity of libration at any time. We are now to consider this effect of the librating force in the direction in which the force is exerted, *viz.* in a meridian analogous to PEAQ in Figure 7.

22. Let FLGA (Fig. 11.) represent the ecliptic on the sphere, S the sun's place in it, L the first point of Libra and A that of Aries; LBA the position of the equator when the sun is at S, and SB the sun's declination. Let FBG be the position into which the equator is pressed in the time i, by a combination of the librating force and the diurnal

revolution; or, which comes to the same, let the spherical angle FBL or ABG be equal to $360 \times \frac{3ti}{2T^2} \times mn \times \frac{a^3-e^3}{a^3}$. Let $h = \frac{a^3-e^3}{a^3}$. Then by spherical Trigonometry, sin. F : sin. BL : : sin. B : sin. FL; and therefore, as FL and the angle FBL are extremely small, the momentary precession $FL = 360 \times \frac{3ti}{2T^2} \times hmn \times \frac{\text{sin. BL}}{\text{sin. F}}$.

23. If FG be bisected in C, and the arc CE be perpendicular to FG, meeting LBA in E, and FBG in D; then the arc CD is the measure of the angle at F. Also, as FL is extremely small, CE may be taken for the measure of the angle at L. Hence as BDE is a right angle, radius : sin. BE or cos. BL : : sin. EBD : sin. ED. Consequently as ED is extremely small, $360 \times \frac{3ti}{2T^2} \times hmn \times \frac{\text{cos. BL}}{\text{radius}} = \text{ED}$, the momentary nutation, or the momentary change in the inclination of the equator to the ecliptic.

From the last proportion, and that concluding the preceding article, it follows that $FL : ED : : \frac{\text{sin. BL}}{\text{sin. F}} : \frac{\text{cos. BL}}{\text{radius}} : : \frac{\text{radius} \times \text{sin. BL}}{\text{cos. BL}}$: sin. F. But cos. BL : radius : : sin. BL : $\frac{\text{radius} \times \text{sin. BL}}{\text{cos. BL}} = \text{tang. BL}$; and therefore, the momentary precession FL is to the momentary nutation ED, as the tangent of the right ascension BL to the sine of the obliquity of the ecliptic.

24. Let $b =$ the sine of the obliquity of the ecliptic, $c =$ its cosine; $x =$ the arc LS, $x =$ its sine, and $y =$ its cosine; and let $2p =$ the circumference of the ecliptic. Then as LBS is a right angle, by the circular parts, cos. BS $\times$ cos. BL $=$ radius $\times$ cos. LS; that is $n \times$ cos. BL $= y$, radius being 1. Again, radius : x : : $b : bx =$ the sine of BS $= m$. Consequently, cos. BS $\times$ sin. BS $\times$ cos. BL $= mn \times$ cos. BL $= bxy$; and therefore,

by the preceding article, the momentary nutation is $360 \times \frac{3i}{2T^2} \times hbxy$, radius being 1.

Again, $2p : \dot{x} :: T : \frac{T\dot{x}}{2p} = \dot{t}$. Also $\sqrt{1-xx} = y$, and, by the fluxional doctrine of circular arcs, $\dot{z} = \frac{\dot{x}}{\sqrt{1-xx}}$; and therefore $\dot{t} = \frac{T\dot{z}}{2p\sqrt{1-xx}}$. These values of $\dot{t}$ and y being put for them in the above expression, the momentary nutation, or, which is the same thing, the fluxion of the nutation is $360 \times \frac{3i}{2T} \times \frac{bbx\dot{z}}{2p}$. Consequently the nutation, when the sun is at S, is $360 \times \frac{3i}{4T} \times \frac{bbxx}{2p}$.

When the sun arrives at the solstitial point C, then x becomes equal to 1, and the nutation is then $360 \times \frac{3i}{4T} \times \frac{bb}{2p} = 180 \times \frac{3i}{4T} \times \frac{a^2-e^2}{a^2} \times \frac{b}{p}$ in degrees, or $10800 \times \frac{3i}{4T} \times \frac{a^2-e^2}{a^2} \times \frac{b \times 60}{p}$ in seconds. Now $t =$ one sidereal day, $T = 366\frac{1}{4} = \frac{1465}{4}$, and therefore $4T = 1465$. According to Sir ISAAC NEWTON's determination of the figure of the earth, a is as 231, e as 230, and therefore $\frac{a^2-e^2}{a^2} = \frac{461}{53361}$. Also supposing the obliquity of the ecliptic 23° 27′ 45″, $b = .3981487$, and $p = 3.14159265$. Consequently $10800 \times \frac{3i}{4T} \times \frac{a^2-e^2}{a^2} \times \frac{b \times 60}{p} = 10800 \times \frac{3}{1465} \times \frac{461}{53361} \times \frac{23.888922}{3.14159265}$, the computation of which may be finished in the following manner.

10800	Log.	4.0334238	1465	Log. - -	3.1658876
3	Log.	0.4771213	53361	Log. - -	4.7272240
461	Log.	2.6637009	3.14159265	Log.	0.4971499
23.888922	Log.	1.3781998			8.3902115
		8.5524458			
		8.3902115			
		0.1622343 = Log. of 1″.4529, and there-			

fore the nutation caused by the action of the sun in a quarter of a year is 1″ 27‴ nearly.

25. By the circular parts, radius $\times c =$ cot. LS $\times$ tang. BL $= c$, radius being 1. But $x : \sqrt{1-xx} :: 1 : \frac{\sqrt{1-xx}}{x} =$ cot. LS; and therefore $\frac{cx}{\sqrt{1-xx}} =$ tang. BL. Consequently by the last and article 23, $b : \frac{cx}{\sqrt{1-xx}} :: 360 \times \frac{3l}{2\Gamma} \times \frac{bbxx}{2p} : 360 \times \frac{3cl}{4\Gamma} \times \frac{b}{p} \times \frac{x^2\dot{x}}{\sqrt{1-xx}} =$ the fluxion of the precession when the sun is at S. Now the fluent of $\frac{x^2\dot{x}}{\sqrt{1-xx}}$ is $\frac{x-x\sqrt{1-xx}}{2}$. For $\dot{z} = \frac{\dot{x}}{\sqrt{1-xx}}$, and the fluxion of $x\sqrt{1-xx}$ is $\dot{x}\sqrt{1-xx} - \frac{x^2\dot{x}}{\sqrt{1-xx}}$. Consequently the whole fluxion of $z - x\sqrt{1-xx}$ is $\frac{\dot{x}}{\sqrt{1-xx}} - \dot{x}\sqrt{1-xx} + \frac{x^2\dot{x}}{\sqrt{1-xx}} = \frac{\dot{x}}{\sqrt{1-xx}} - \frac{\dot{x}}{\sqrt{1-xx}} + \frac{x^2\dot{x}}{\sqrt{1-xx}} + \frac{x^2\dot{x}}{\sqrt{1-xx}} = \frac{2x^2\dot{x}}{\sqrt{1-xx}}$. Consequently the fluent of the precession, when the sun is at S, is $360 \times \frac{3cl}{4\Gamma} \times \frac{b}{p} \times \frac{z-x\sqrt{1-xx}}{2}$.

When the sun arrives at the solstitial point C, then x becomes equal to 1, and z becomes equal to $\frac{p}{2}$, and the quantity of the precession is then $360 \times \frac{3cl}{4\Gamma} \times \frac{b}{p} \times \frac{p}{4} = 360 \times \frac{3l}{4\Gamma} \times \frac{cb}{4}$. This expressed in numbers, admitting the obliquity of the ecliptic to be 23° 27′ 45″, is $90 \times \frac{3}{1465} \times \frac{461}{53361} \times .9173813$ in degrees, and the same in seconds is $5400 \times \frac{3}{1465} \times \frac{461}{53361} \times 55.042878$. This calculation may be finished in the following manner.

5400	Log. 3.7328938	1465 -	Log. 3.1658376	
8	Log. 0.4771213	53361 -	Log. 4.7272240	
461	Log. 2.6637009		7.8930616	
55.042878	Log. 1.7407010			
	8.6139170			
	7.8930616			
	0.7208554 = Log. of 5".2584.			

Consequently the annual precession, caused by the disturbing force of the sun, is 21".0336.

The obliquity of the ecliptic has been assumed as equal to 23° 27′ 45″, such being its magnitude, very nearly, at the beginning of the year 1807.

From the general expression $360 \times \frac{yl}{4T} \times \frac{bh \times x}{zp}$, obtained in article 24, it is evident, that when the sun is in either of the equinoctial points, the nutation becomes equal to 0. Supposing therefore the earth to be subject to no other disturbing force than that of the sun, at each of the equinoxes the earth's diurnal revolution is made about its axis of figure, as PL in Fig. 9; but as at other times the disturbing force tends to cause a libration about a diameter of the equator, it is evident from article 10, that the axis about which it revolves deviates, by a quantity extremely small, from its axis of figure. A similar deviation, of the axis of revolution from the axis of figure, is produced by the action of the moon; but a minute examination of these deviations is foreign to the present design. As the foregoing articles extend beyond the supposed difficulty in the subject, it is deemed unnecessary either to add to their number, or to lengthen this Paper by such additional remarks, as may be met with in every respectable publication on Physical Astronomy.

Fig. 2.

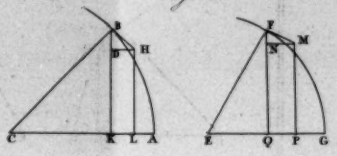


Fig. 7.

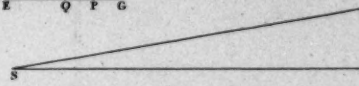


Fig. 3.

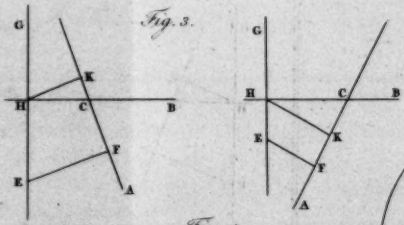


Fig. 6.

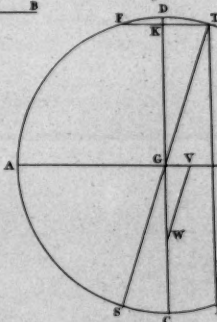


Fig. 4.

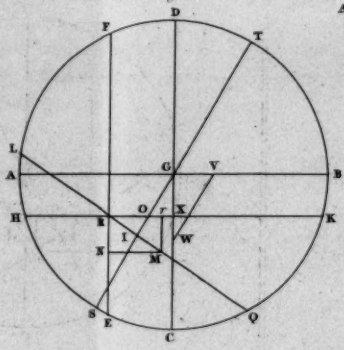


Fig.

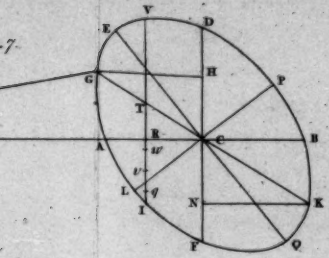


Fig. 5.

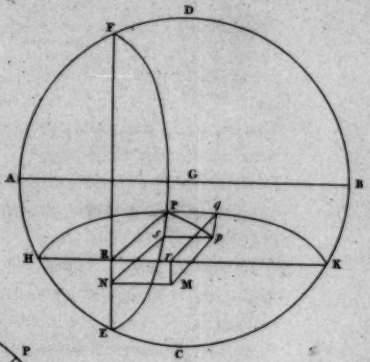


Fig. 8.

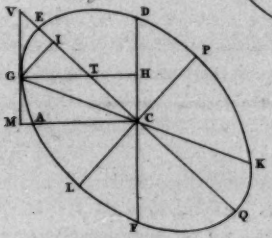


Fig. 10.

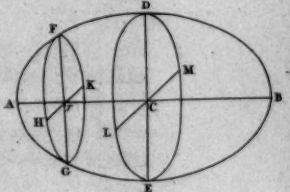


Fig. 9.

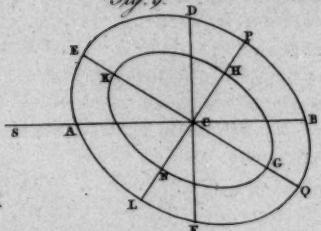
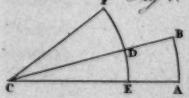


Fig. 11.

Fig. 1.



Basire sc.

III. *An Account of two Children born with Cataracts in their Eyes, to shew that their Sight was obscured in very different Degrees; with Experiments to determine the proportional Knowledge of Objects acquired by them immediately after the Cataracts were removed.* *By* Everard Home, *Esq. F. R. S.*

Read January 15, 1807.

MR. CHESELDEN'S observations on this subject, recorded in the Phil. Trans. for the year 1728, pointed out two material facts; that vision alone gives no idea of the figure of objects, or their distance from the eye, since a very intelligent boy, 13 years of age, upon recovering his sight was unable to distinguish the outline of any thing placed before him, and thought that every object touched his eye.

Mr. WARE'S cases, which have also a place in the Phil. Trans. for 1801, and are compared with that of Mr. CHESELDEN, appear to lead to a different conclusion. The following observations are laid before the Society with a view to explain this circumstance.

CASE I.

WILLIAM STIFF, twelve years of age, was admitted into St. George's Hospital under my care, on the 17th of July, 1806, with cataracts in his eyes, which, according to the account of his mother, existed at the time of birth. From earliest

infancy he never stretched out his hand to catch at any thing, nor were his eyes directed to objects placed before him, but rolled about in a very unusual manner, although in other respects he was a lively child. The eyes were not examined till he was six months old, and at that time the cataracts were as distinct as when he was received into the hospital.

Previous to an operation being performed, the following circumstances were ascertained respecting his vision. He could distinguish light from darkness, and the light of the sun from that of a fire or candle: he said it was redder, and more pleasant to look at, but lightning made a still stronger impression on his eyes. All these different lights he called red. The sun appeared to him the size of his hat. The candle flame was larger than his finger, and smaller than his arm. When he looked at the sun he said it appeared to touch his eye. When a lighted candle was placed before him both his eyes were directed towards it, and moved together. When it was at any nearer distance than 19 inches, he said it touched his eyes. When moved further off he said it did not touch them; and at 22 inches it became invisible.

On the 21st of July the operation of extracting the crystalline lens was performed on the left eye. The capsule of the lens was so very strong as to require some force to penetrate it. When wounded, the contents, which were fluid, rushed out with great violence. Light became very distressing to his eye, and gave him pain. After allowing the eye-lids to remain closed for a few minutes, and then opening them, the pupil appeared clear, but he could not bear exposure to light. On my asking him what he had seen, he said, "your head, which

seemed to touch my eye:" but he could not tell its shape. He went to bed, and took an opiate draught: the pain in his eye lasted about an hour, after which he fell asleep. The whole of that day the light was distressing to his eye, so that he could not bear the least exposure to it.

On the 22d the eye-lids were opened to examine the eye. The light was less offensive. He said he saw my head, which touched his eye. There was so much inflammation on the eye-ball, that a leech was applied to the temple, and the common means for removing inflammation were used.

On the 23d the eye was less inflamed, and he could bear a weak light. The pupil was of an irregular figure, and the wounded cornea had not united with a smooth surface. He said he could see several gentlemen round him, but could not describe their figure. My face, while I was looking at his eye, he said was round and red.

On the 25th the inflammation had subsided, but on the 27th returned, and continued notwithstanding different means were employed for its removal, till the 1st of August, when it was almost entirely gone. On the 4th the eye was apparently so well that an attempt was made in the presence of Mr. Cavendish and Dr. Wollaston to ascertain its powers of vision; but it was so weak that it became necessary to shade the glare of light by hanging a white cloth before the window. The least exertion fatigued the eye, and the cicatrix on the cornea, to which the iris had become attached, drew it down so as considerably to diminish the pupil. From these circumstances nothing could be satisfactorily made out respecting the boy's vision. On the 11th a second attempt was made in the presence of Mr. Cavendish, but the pupil conti-

nued so contracted and irregular, and the eye so imperfect in its powers, that it became necessary a second time to postpone any experiments.

On the 16th of September the right eye was couched. This operation was preferred after what had happened to the other eye, in the hope that there would not be the same degree of inflammation, and as the former cataract was fluid, there was every reason to believe that couching would in this instance be most efficacious.

The operation gave pain, and the light was so distressing to his eye that the lids were closed as soon as it was over, and he was put to bed. The consequent inflammation was not severe, but as soon as the fluid cataract, which had been diffused through the aqueous humour was absorbed, the capsule of the lens was found to be opaque, and the sight consequently imperfect. The eyes were not examined with respect to their vision till the 13th of October, during which period the boy remained quiet in the hospital. On that day the upper part of the pupil of the left eye had in some measure recovered its natural state, and had become transparent, but the cicatrix in the cornea was more extensively opaque than before. The light now was not distressing to either eye, and when strong, he could readily discern a white, red, or yellow colour, particularly when bright and shining. The sun and other objects did not now seem to touch his eyes as before, they appeared to be at a short distance from him. The eye, which had been couched, had the most distinct vision of the two, but in both it was imperfect. The distance at which he saw best was five inches.

When the object was of a bright colour, and illuminated

by a strong light, he could make out that it was flat and broad; and when one corner of a square substance was pointed out to him, he saw it, and could find out the other, which was at the end of the same side, but could not do this under less favourable circumstances. When the four corners of a white card were pointed out, and he had examined them, he seemed to know them: but when the opposite surface of the same card, which was yellow, was placed before him, he could not tell whether it had corners or not; so that he had not acquired any correct knowledge of them, since he could not apply it to the next coloured surface, whose form was exactly the same, with that, the outline of which the eye had just been taught to trace.

CASE II.

JOHN SALTER, seven years of age, was admitted into St. George's Hospital on the 1st of October, 1806, under my care, with cataracts in both eyes, which according to the accounts of his relations had existed from his birth.

After he was received into the hospital, the following circumstances were ascertained respecting his vision. The pupils contracted considerably when a lighted candle was placed before him, and dilated as soon as it was withdrawn. He was capable of distinguishing colours with tolerable accuracy, particularly the more bright and vivid ones.

On the 6th of October the left eye was couched. This operation was preferred to extraction, from a belief that the cataracts were not solid, and as the injury done to the capsule by the operation would be less, there was not the same chance of inflammation, the disposition for which, had been so

strong in the former case. As the eye was not irritable, and was likely to be but little disturbed by this operation, every thing was previously got ready for ascertaining his knowledge of objects, as soon as the operation was over, should the circumstances prove favourable. The operation was attended with success, and gave very little pain. The eye was allowed ten minutes to recover itself: a round piece of card of a yellow colour, one inch in diameter, was then placed about six inches from it. He said immediately that it was yellow, and on being asked its shape, said, "Let me touch "it, and I will tell you." Being told that he must not touch it, after looking for some time, he said it was round. A square blue card, nearly the same size, being put before him, he said it was blue and round. A triangular piece he also called round. The different colours of the objects placed before him he instantly decided on with great correctness, but had no idea of their form. He moved his eye to different distances, and seemed to see best at 6 or 7 inches. His focal distance has been since ascertained to be 7 inches. He was asked whether the object seemed to touch his eye, he said "No;" but when desired to say at what distance it was, he could not tell. These experiments were made in the theatre of the hospital, in which the operation was performed, before the surgeons and all the students. He was highly delighted with the pleasure of seeing, and said it was "so pretty," even when no object was before him, only the light upon his eye. The eye was covered, and he was put to bed, and told to keep himself quiet, but upon the house-surgeon going to him half an hour afterwards, his eye was found uncovered, and he was looking at his bed curtains, which were close drawn.

The bandage was replaced, but so delighted was the boy with seeing, that he again immediately removed it. This circumstance distressed the house-surgeon, who had been directed to prevent him from looking at any thing till the next day, when the experiment was to be repeated. Finding that he could not enforce his instructions, he thought it most adviseable to repeat the experiment about two hours after the operation. At first the boy called the different cards round; but upon being shewn a square, and asked if he could find any corners to it, he was very desirous of touching it. This being refused, he examined it for some time, and said at last that he had found a corner, and then readily counted the four corners of the square; and afterwards when a triangle was shewn him, he counted the corners in the same way; but in doing so his eye went along the edge from corner to corner, naming them as he went along.

Next day, when I saw him, he told me he had seen "the "soldiers with their fifes and pretty things." The guards in the morning had marched past the hospital with their band; on hearing the music he had got out of bed, and gone to the window to look at them. Seeing the bright barrels of the musquets, he must in his mind have connected them with the sounds which he heard, and mistaken them for musical instruments. On examining the eye 24 hours after the operation, the pupil was found to be clear. A pair of scissors was shewn him, and he said it was a knife. On being told he was wrong, he could not make them out; but the moment he touched them he said they were scissors, and seemed delighted with the discovery. On being shewn a guinea at the distance of 15 inches from his eye, he said it was a seven

shilling piece, but placing it about 5 inches from his eye, he knew it to be a guinea; and made the same mistake, as often as the experiment was repeated.

From this time he was constantly improving himself by looking at, and examining with his hands, every thing within his reach, but he frequently forgot what he had learnt. On the 10th I saw him again, and I told him his eye was so well that he might go about as he pleased without leaving the room. He immediately went to the window, and called out, "What is that moving?" I asked him what he thought it was? He said, "A dog drawing a wheelbarrow. There is one, two, three dogs drawing another. How very pretty!" These proved to be carts and horses on the road, which he saw from a two pair of stairs window.

On the 19th, the different coloured pieces of card were separately placed before his eye, and so little had he gained in thirteen days, that he could not without counting their corners one by one tell their shape. This he did with great facility, running his eye quickly along the outline, so that it was evident he was still learning, just as a child learns to read. He had got so far as to know the angles, when they were placed before him, and to count the number belonging to any one object.

The reason of his making so slow a progress was, that these figures had never been subjected to examination by touch, and were unlike any thing he was accustomed to see.

He had got so much the habit of assisting his eyes with his hands, that nothing but holding them could keep them from the object.

On the 26th the experiments were again repeated on the

couched eye, to ascertain the degree of improvement which had been made. It was now found that the boy, on looking at any one of the cards in a good light, could tell the form nearly as readily as the colour.

From these two cases the following conclusions may be drawn:

That, where the eye before the cataract is removed, has only been capable of discerning light, without being able to distinguish colours, objects after its removal will seem to touch the eye, and there will be no knowledge of their outline; which confirms the observations made by Mr. CHESELDEN:

That where the eye has previously distinguished colours, there must also be an imperfect knowledge of distances, but not of outline, which however will afterwards be very soon acquired, as happened in Mr. WARE's cases. This is proved by the history of the first boy in the present Paper, who before the operation had no knowledge of colours or distances, but after it, when his eye had only arrived at the same state, that the second boy's was in before the operation, he had learnt that the objects were at a distance, and of different colours: that when a child has acquired a new sense, nothing but great pain or absolute coercion, will prevent him from making use of it.

In a practical view, these cases confirm every thing, that has been stated by Mr. POTT and Mr. WARE, in proof of cataracts in children being generally soft, and in favour of couching, as being the operation best adapted for removing them. They also lead us to a conclusion of no small importance, which has not before been adverted to; that when the cataract has assumed a fluid form, the capsule, which is

naturally a thin transparent membrane, has to resist the pressure of this fluid, which like every other diseased accumulation is liable to increase, and distend it, and therefore the capsule is rendered thicker and more opaque in its substance, like the coats of encysted tumours in general.

As such a change is liable to take place, the earlier the operation is performed in all children, who have cataracts completely formed, the greater is their chance of having distinct vision after the operation. It is unnecessary to point out the advantages to be derived from its being done at a more early age, independent of those respecting the operation itself.

IV. *Observations on the Structure of the different Cavities, which constitute the Stomach of the Whale, compared with those of ruminating Animals, with a View to ascertain the Situation of the digestive Organ.* By Everard Home, *Esq. F. R. S.*

Read February 12, 1807.

THE following observations are in some measure a continuation of those upon the stomachs of ruminating animals contained in a former Paper. They are intended to show that the stomach of the whale forms a link in the gradation towards the stomachs of truly carnivorous animals.

This subject was brought under my consideration by the following circumstances. While at Worthing, on the Sussex coast, in the month of August last, a Delphinus Delphis of LINNÆUS, or small bottle-nose whale of Mr. HUNTER, was brought on shore by the fishermen alive. I immediately purchased it, with a view of enriching the HUNTERIAN collection with the skeleton, and other parts of its structure.

The stomach was the particular object of my own attention; for, having been so lately employed in considering the stomachs of ruminating animals, I was pleased with an opportunity of examining in a recent state the stomach of one of the whale tribe, to which the porpoise belongs, with a view to ascertain more accurately than had been hitherto done, the real resemblance between its structure, and that of the stomachs of ruminating animals.

The structure of the stomach of one species of whale was not new to me, having twenty years ago assisted Mr. HUNTER in dissecting the piked whale, but at that time I only viewed the different parts of its structure with the eye of a common observer, while now my mind was particularly directed to the peculiarities of the stomach. In this examination I discovered a resemblance between the second, third, and fourth cavities in the whale, and the two portions of the fourth cavity in the bullock and camel, which appears to throw some light upon the uses of those parts, as well as upon digestion in general.

As in the former Paper a particular description was given of the stomach of the bullock and camel, as examples of ruminants with and without horns, it will be proper here to describe the stomach of the bottle-nose porpoise, as an example of the whale tribe.

In the bottle-nose porpoise the œsophagus is very wide, has a number of longitudinal folds, and is lined with a strong white cuticle, which is continued over the internal surface of the first stomach.

The first stomach lies in the direction of the œsophagus, which is continued into it, there being no contraction to mark its origin. It is of an oval form, and bears a strong resemblance in shape to a Florence flask. The cavity is 15 inches in length, and 9 in diameter. The internal surface has a very corrugated appearance, and its cuticular covering is thick and strong. The coats of the cavity are firm, and its bottom is surrounded by a strong muscular covering.

The orifice, which leads to the second stomach, is at right angles to the cavity, and is situated a little way below the

termination of the œsophagus. It is surrounded by several semicircular doublings of the internal membrane: the broadest of these is on the lower part, these are thick, and appear to be glandular.

There is a canal between the first and second cavities 3 inches long, which opens into the second by a projecting orifice, and the cuticular covering of the first stomach terminates immediately beyond this orifice, which is 2½ inches in diameter.

This second stomach is nearly spherical, about 7 inches in diameter. Its internal surface has a honeycombed appearance, formed by soft ridges of a glandular structure, leaving interstices of some depth between them. This structure gives the coats a considerable degree of thickness.

The opening into the third stomach is almost close to that which enters the second, and is only ½ of an inch in diameter.

The third cavity is nearly spherical, and is 2 inches in diameter. Its internal surface is smooth, and there are every where small orifices of ducts of glands opening into its cavity. The aperture, which communicates between this and the fourth stomach is ½ of an inch in diameter.

The fourth cavity is nearly cylindrical like an intestine, but rather widest at its furthest extremity. It is 14½ inches long; its greatest diameter is 3 inches. The internal membrane is smooth, and for 3 inches towards its origin, and 4 inches towards its termination has numerous orifices through which secretions are poured into the cavity. The pylorus, which is the boundary of this stomach, is a round orifice ⅝ of an inch in diameter.

Immediately beyond the pylorus there is a dilatation of the gut, which both CUVIER and HUNTER call a cavity belonging to the stomach. It must however be considered as duodenum, since the common duct of the liver and pancreas opens into it; the longitudinal *valvulæ conniventes* have their origin in it; and there is no transverse constriction any where beyond it, to mark the beginning of an intestine. Such an enlargement of the duodenum is very common in other animals, and has been described in the account of the camel. The coats of this portion of the duodenum are thicker than those of the fourth stomach. The annexed drawings (Plate III.) will give a better idea of these different parts than can be conveyed by any verbal description.

The number of cavities constituting the stomach are not the same in all animals of the whale tribe. In the common porpoise, grampus, and piked whale, the number is the same as in the bottle-nose porpoise; but in the bottle-nose whale of Dale there are two more cavities. This variation is however by no means material, since the general structure of the stomach is the same.

In all of the whale tribe there is one cavity lined with a cuticle, as in the bullock and camel.

In all of them there is a second cavity made up of a very glandular structure. In the porpoise, grampus, and large bottle-nose whale this structure resembles that which is above described. In the piked whale the rugæ are longitudinal and deep, but in some places united by cross bands; and as the piked whale has whalebone teeth, the great whalebone whale will probably, from the analogy of its teeth, resemble it in the structure of its stomach.

The third cavity in all of them is very small, and bears a strong resemblance to the third cavity in the camel's stomach; its use, therefore, is probably the same.

The fourth stomach in all of them has a smooth internal surface, with the orifices of glands opening into its cavity. In the bottle-nose whale of Dale the two additional cavities have the same internal structure, and therefore must have the same general use, with a greater extension of surface, and the subdivisions will make the food pass more slowly into the intestine.

The first stomach of the whale is not only a reservoir, but the food undergoes a considerable change in it. The flesh is entirely separated from the bones in this cavity, which proves that the secretion from the glandular part has a solvent power. This was found to be the case in the bottle-nose porpoise and large bottle-nose whale. In both of them several handfuls of bones were found in the first stomach, without the smallest remains of the fish, to which they belonged. The soft parts only can be conveyed into the second and third stomachs, the orifices being too small to admit the bones to pass.

The bones must therefore be reduced to a jelly in the first stomach, and although the process, by which this is effected, being slower than that, which separates the flesh, is the reason of their being found in such quantity in the cavity, the means by which it is performed are probably the same.

The second cavity was supposed by Mr. HUNTER to be the true digesting stomach, in which the food becomes chyle, and the use of the third and fourth he looked upon as not exactly ascertained.*

* *Vide* Observations on the Structure and Œconomy of Whales. By JOHN HUNTER. Phil. Trans. Vol. LXXVII. page 411.

Upon what ground Mr. Hunter was led to draw this conclusion cannot now be ascertained; and, such is my respect for his opinion, that nothing but the following observations, supported by facts, could lead me to form a different one. In considering this subject, it struck me that the second stomach, could not be that, in which chyle is formed, since that process having been completed, any other cavities would be superfluous. The last cavity in all stomachs is that, in which the process must be brought to perfection: and therefore the most essential change, which the food undergoes, or that, by which it is formed into chyle should be performed in that cavity. Surveying the different cavities, in the whale's and ruminating stomachs with this impression on my mind, and comparing them with the single stomachs of carnivorous animals, it appeared that the first point, which required to be ascertained was, which of the cavities in these more complex stomachs bears the greatest resemblance to the simple one. The fourth of the whale is certainly more like the human stomach than the second or third. I therefore concluded that the fourth, both from analogy and situation, is the stomach in which the process is completed: and that in this animal, from the peculiarities of its economy, and the nature of the food, not only a cuticular stomach is necessary, but also two glandular ones, in which it undergoes changes preparatory to its being converted into chyle.

Having satisfied myself upon this subject, and having compared the stomachs of the whale, with the fourth of the camel, the contraction or partial division of the camel's, made it apparent that the lower portion only of that cavity, which resembles in shape, and internal appearance the human

stomach, is the cavity in which chyle is formed, and the upper or plicated portion is only to prepare the food, and is therefore analogous to the second in the whale.

As the same appearances are met with in the fourth stomach of the bullock, as well as in the camel, although there is no permanent contraction, or division between them, the upper or plicated portion must be considered as a preparatory organ, and the lower portion as that, in which the formation of chyle is completed. This receives further confirmation from a more attentive examination of the parts, immediately after death, by which it was found that before the stomach has been disturbed there is an evident muscular contraction between the plicated and lower portion. This appearance was met with in every instance that was examined, and these were not fewer than nine or ten. Added to this the lower portion, on a more minute inspection, has an appearance somewhat similar to the inner membrane of the human stomach: and the surface of the plicæ is in many respects different.

From the facts and observations which have been stated, it appears that in many animals of the class Mammalia, the food undergoes different changes preparatory to its being converted into chyle, and this last process is effected by a somewhat similar secretion, since the part of the stomach which produces it, has in all of them an evident similarity of structure.

The above facts appear to throw some light on the digestion of the different kinds of food, and open a wide field of enquiry into one of the most interesting parts of the animal œconomy, which has been hitherto too much neglected. In the present very limited state of our knowledge there are

many circumstances which cannot be accounted for: these however, will be explained when a further progress has been made in this investigation.

It is obvious, that as the stomachs of carnivorous animals are the most simple, animal substances, on which they feed, require a shorter process to convert them into chyle than vegetables; but why the whale tribe, which live on fish, should have a more complex stomach, it is not easy to explain: since fish are very readily converted into chyle, in the stomachs of animals of their own class, as well as in the human stomach, and there is therefore reason to believe that they require as little preparation for that process, if not less than animal substances.

The fish bones swallowed by the whale tribe being retained in the cuticular bag, till they are reduced to jelly, explains the circumstance of cows, and other ruminating animals being able occasionally to live on fish, (a fact, of which there is no doubt, both in the Orkneys and in Iceland,) since, if the bones are dissolved in the paunch, the other stomachs, are in no danger of being injured from the animal living on this kind of food.

Whether these cavities, which I have called preparatory stomachs, are solely for purposes connected with digestion, or are also in any way connected with the formation of secretions peculiar to those animals, cannot be ascertained in the present state of our knowledge of digestion.

The oil of the physeter, which crystallizes into spermaceti, shews some affinity in this respect to the secretion of fat, that becomes suet, which is only met with in ruminating animals: but on the other hand, the oil of the rest of the

whale tribe does not form this substance, more than the fat of the horse produces tallow. These facts may be afterwards explained by an examination of the digestive organs of the physeter, when an anatomist shall have an opportunity of examining them.

These are enquiries which do not belong to the present Paper, as it is only intended to add some facts to those already laid before the Society, and in a future communication I hope still further to increase their number.

EXPLANATION OF THE PLATES.

(PLATE III.)

This plate represents the first cavity of the stomach of the bottle-nose porpoise laid open to shew its internal structure.

a a. The œsophagus lined with cuticle.

b b. The first cavity of the stomach, also lined with cuticle.

c c. The glandular structure forming folds round the orifice leading to the second cavity, also lined with cuticle.

(PLATE IV.)

This plate represents the internal surface of the second, third, and fourth cavities of the stomach.

a a. The outside of the first cavity to shew its external form.

b b. The inner surface of the second cavity, made up of a honey-combed structure, composed of soft membraneous folds, which have no cuticular covering.

c c. The third cavity.

d d. The fourth cavity.

e e. The orifices of excretory ducts of glands.

f f. The enlargement of the substance immediately beyond the pylorus, into which the common duct from the liver and pancreas opens.

g. The opening of the common duct.

SCALE. Six Inches to a Foot.

W. Clift del.

Bauer sc.

SCALE. Five Inches to a Foot

IV. *On the Formation of the Bark of Trees. In a Letter from T. A. Knight, Esq. F. R. S. to the Right Honourable Sir Joseph Banks, K. B. P. R. S. &c.*

Read February 19, 1807.

My Dear Sir,

An extraordinary diversity of opinion appears to have prevailed among naturalists, respecting the production and subsequent state of the bark of trees.

According to the theory of Malpighi, the cortical substance, which is annually generated, derives its origin from the older bark: and the interior part of this new substance is annually transmuted into alburnum, or sap wood; whilst the exterior part, becoming dry and lifeless, forms the exterior covering, or cortex.

The opinions of Grew do not appear to differ much from those of Malpighi: but he conceives the interior bark to consist of two distinct substances, one of which becomes alburnum, whilst the other remains in the state of bark: he, however, supposes the insertments in the wood, the "utriculi" of Malpighi, and the "tissu cellulaire" of Du Hamel, to have originally existed in the bark.

Hales on the contrary contends, that the bark derives its existence from the alburnum, and that it does not undergo any subsequent transformation.

The discoveries of Du Hamel have thrown much light

on the subject; but his experiments do not afford any conclusive result, and some of them may be adduced in support of either of the preceding hypotheses: and a modern writer (MIRBEL*) has endeavoured to combine and reconcile, in some degree, the apparently discordant theories of MALPIGHI and HALES. He contends with HALES, that the alburnum gives existence to the new layer of bark; but that this bark subsequently changes into alburnum, though not precisely in the manner described by MALPIGHI.

So much difference of opinion, amongst men so capable of observing, sufficiently evinces the difficulty of the subject they endeavoured to investigate; and in a course of experiments, which has occupied more than twenty years, I have scarcely felt myself prepared, till the present time, even to give an opinion respecting the manner, in which the cortical substance is generated in the ordinary course of its growth, or reproduced, when that, which previously existed, has been taken off.

DU HAMEL has shewn, that the bark of some species of trees is readily reproduced, when the decorticated surface of the alburnum is secluded from the air; and I have repeated similar experiments on the apple, the sycamore, and other trees, with the same result: I have also often observed a similar reproduction of bark on the surface of the alburnum of the *Wych elm* (Ulmus montana) *in shady situations*, when no covering whatever was applied. A glareous fluid, as DU HAMEL has stated, exudes from the surface of the alburnum: this fluid appears to change into a pulpous unorganised mass, which subsequently becomes organised and cellular; and the

* Traité d'Anatomie et de Physiologie végétales.

matter, which enters into the composition of this cellular substance, is evidently derived from the alburnum.

These facts are therefore extremely favourable to the theory of HALES; but other facts may be adduced which are scarcely consistent with that theory.

The internal surface of pieces of bark, when detached from contact with the alburnum, provided they remain united to the tree at their upper ends, much more readily generate a new bark, than the alburnum does under similar circumstances: a similar fluid exudes from the surfaces of both, and the same phenomena are observable in both cases. The cellular substance, however, which is thus generated, though it presents every external appearance of a perfect bark, is internally very imperfectly organized; and the vessels which contain the true sap in the bark, are still wanting; and I have found, that these may be made, by appropriate management, to traverse the new cellular substance in almost any direction. When I cut off all communication above, and on one side, between the old bark and that substance, I observed, that the vessels proceeded across it, from the old bark on the other side, taking always in a greater or less degree an inclination downwards; and when the cellular substance remained united to the bark at its upper end only, the vessels descended nearly perpendicularly down it; but they did not readily *ascend* into it, *when it was connected with the bark at its lower extremity only*; the result of similar experiments, when made on different species of trees, was, however, subject to some variations.

Pieces of bark of the walnut-tree, which were two inches

broad, and four long, having been detached from contact with the alburnum, except at their upper ends, and covered with a plaister composed of bees-wax and turpentine, in some instances, and with clay only in others, readily generated the cellular substance of a new bark; and between that and the old detached bark, very nearly as much alburnum was deposited as in other parts of the tree, where the bark retained its natural position; which, I think, affords very decisive evidence of the descent of the sap through the bark. Similar pieces of bark, under the same mode of treatment, but united to the tree at their lower ends only, did not long remain alive, except at their lower extremities; and there a very little alburnum only was generated. Other pieces of bark of the same dimensions, which were laterally united to the tree, continued alive almost to their extremities; and a considerable portion of alburnum was generated, particularly near their lower edges; the sap appearing in its passage across the bark to have been given a considerable inclination downwards: probably owing to an arrangement in the organization of the bark, that I have noticed in a former memoir,* which renders it better calculated to transmit the sap towards the roots than in any other direction.†

I have in very few instances been able to make the walnut-tree reproduce its bark from the alburnum, though under the same management I rarely failed to succeed with the sycamore and apple-tree. Pieces of the bark of the apple-tree will also live, and generate a small portion of alburnum, though only attached to the tree at their lower extremities;

* Philosophical Transactions of 1804.

probably owing to a small part of the true sap being carried upwards by capillary attraction, when the proper action of the cortical vessels is necessarily suspended.

The preceding experiments, and the authority of Du Hamel, having perfectly satisfied me, that both the alburnum and bark of trees are capable of generating a new bark, or at least of transmitting a fluid capable of generating a cellular substance, to which the bark in its more perfectly organized state owes its existence, my attention was directed to discover the sources from which this fluid is derived. Both the bark and the alburnum of trees are composed principally of two substances; one of which consists of long tubes, and the other is cellular; and the cellular substance of the bark is in contact with the similar substance in the alburnum, and through these I have long suspected the true sap to pass from the vessels of the bark to those of the alburnum.* The intricate mixture of the cellular and vascular substances long baffled my endeavours to discover from which of them, in the preceding cases, the sap, and consequently the new bark, proceeded; but I was ultimately successful.

The cellular substance, both in the alburnum and bark of old pollard oaks, often exists in masses of near a line in width, and this organization was peculiarly favourable to my purpose. I therefore repeated on the trunks of trees of this kind, experiments similar to those above-mentioned which were made on the walnut-tree.

Apparently owing to the small quantity of sap, which the old pollard trees contained, their bark was very imperfectly reproduced; but I observed a fluid to ouze from the cellular

* Phil. Trans. 1805, page 14.

substance, both of the bark and alburnum; and on the surface of these substances alone, in many instances, the new bark was reproduced in small detached pieces.

I have endeavoured to prove in former communications,* that the true sap of trees acquires those properties which distinguish it from the fluid recently absorbed, by circulating through the leaf; and that it descends down the bark, where part of it is employed in generating the new substances annually added to the tree; and that the remainder, not thus expended, passes into the alburnum, and there joins the ascending current of sap. The cellular substance, both of the bark and alburnum, has been proved, in the preceding experiments, to be capable of affording the sap a passage through it; and therefore it appears not very improbable, that it executes an office similar to that of the anastomosing vessels of the animal economy, when the cellular surfaces of the bark and alburnum are in contact with each other; and, when detached, it may be inferred, that the passing fluid will exude from both surfaces: because almost all the vessels of trees appear to be capable of an inverted action in giving motion to the fluids which they carry.

As the power of generating a new bark appeared in the preceding cases to exist alike in the sap of the bark and of the alburnum, I was anxious to discover how far the fluid, which ascends through the central vessels of the succulent annual shoot, is endued with similar powers. Having therefore made two circular incisions through the bark, round the stems of several annual shoots of the vine, as early in the summer as the alburnum within them had acquired sufficient

* Phil. Trans. 1801, 1805, and 1806.

maturity to perform its office of carrying up the sap, I took off the bark between these incisions; and I abraded the surface of the alburnum to prevent a reproduction of it. The alburnum in the decorticated spaces soon became externally dry and lifeless; and several incisions were then made longitudinally through it. The incisions commenced a little above, and extended below the decorticated spaces, so that, if the sap of the central vessels generated a cellular substance (as I concluded it would), that substance might come into contact and form a union with the substance of the same kind emitted by the bark above and below.

The experiment succeeded perfectly, and the cellular substances generated by the central vessels, and the bark, soon united, and a perfect vascular bark was subsequently formed beneath the alburnum, and appeared perfectly to execute the office of that which had been taken off; the medulla appeared to be wholly inactive.

I have already observed, that the vessels, which were generated in the cellular substance on the surface of the alburnum of the sycamore and the apple-tree, traversed that substance in almost every direction; and the same thing appears to occur beneath the old bark, when united to the alburnum. For having attentively examined through every part of the spring and summer, the formation of the internal bark, and alburnous layer beneath it, round the bases of regenerated buds, which I had made to spring from smooth spaces on the roots and stems of trees, I found every appearance perfectly consistent with the preceding observations. A single shoot only was suffered to spring from each root and stem, and from the base of this, in every instance the cortical vessels

dispersed themselves in different directions. Some descended perpendicularly downwards, whilst others diverged on each side, round the alburnum, with more or less inclination downwards, and met on the opposite side of it. The same pulpous and cellular substance appeared to cover the surfaces of the bark and alburnum, when in contact with each other, as when detached; and through this substance the ramifications of the vessels of the new bark extended themselves, appearing to receive their direction from the fluid sap which descended from the bark of the young shoots, and not to be, in any degree, influenced in their course by the direction taken by the cortical and alburnous vessels of the preceding year.

Whenever the vessels of the bark, which proceeded from different points, met each other, an interwoven texture was produced, and the alburnum beneath acquired a similar organisation: and the same thing occurs, and is productive of very important effects, in the ordinary course of the growth of trees. The bark of the principal stem, and of every lateral branch, contains very numerous vessels, which are charged with the descending true sap; and at the juncture of the lateral branch with the stem, these vessels meet each other. A kind of pedestal of alburnum, the texture of which is much interwoven, is in consequence formed round the base of the lateral branch; which thus becomes firmly united to the tree. This pedestal, though apparently a part of the branch, derives a large portion of the matter, annually added to it, from the cortical vessels of the principal stem; and thence, in the event of the death of the lateral branch, it always continues to live. But it not unfrequently happens, that a lateral branch forms a very acute angle with the principal stem, and, in this case,

the bark between them becomes compressed and inactive; no pedestal is in consequence formed, and the attachment of such a branch to the stem becomes extremely feeble and insecure.* Instead of the reproduced buds of the preceding experiment, buds were inserted in the foregoing summer, or attached by grafting in the spring; and, when these succeeded, though they were in many instances taken from trees of different species, and even of different genera, no sensible difference existed in the vessels, which appeared to diverge into the bark of the stock, from these buds and from those reproduced in the preceding experiments.

It appears, therefore, probable, that a pulpous organisable mass first derives its matter either from the bark, or the alburnum; and that this matter subsequently forms the new layer of bark; for, if the vessels had proceeded, as radicles,† from the inserted buds, or grafts, such vessels would have been, in some degree, different from the natural vessels of the bark of the stocks; and it does not appear probable, even without referring to the preceding facts, that vessels should be extended, in a few days, by parts successively added to their

* The advantages, which may be obtained by pruning timber trees judiciously, appear to be very little known. I have endeavoured to ascertain the practicability of giving to trees such forms as will render their timber more advantageously convertible to naval or other purposes. The success of the experiments, on small trees has been complete, and the results perfectly consistent, in every case, with the theory I have endeavoured to support in former memoirs; and I am confident, that by appropriate management, the trunks and branches of growing trees may be moulded into the various forms best adapted to the use of the ship-builder; and that the growth of the trees may at the same time be rendered considerably more rapid, without any expense or temporary loss to the proprietor.

† Darwin's Phytologia.

extremities, from the leaves to the extremities of the roots; which are, in many instances, more than two hundred feet distant from each other. I am, therefore, inclined to believe, that, as the preceding facts seem to indicate, the matter, which composes the new bark, acquires an organisation calculated to transmit the true sap towards the roots, as that fluid progressively descends from the leaves in the spring; but whether the matter, which enters into the composition of the new bark, be derived from the bark or the alburnum, in the ordinary course of the growth of the tree, it will be extremely difficult to ascertain.

It is, however, no difficult task to prove, that the bark does not, in all cases, spring from the alburnum; for many cases may be adduced in which it is always generated previously to the existence of the alburnum beneath it: but none, I believe, in which the external surface of the alburnum exists previously to the bark in contact with it, except when the cortical substance has been taken off, as in the preceding experiments. In the radicle of germinating seeds, the cortical vessels elongate, and new portions of bark are successively added to their points, many days before any alburnous substance is generated in them; and in the succulent annual shoot the formation of the bark long precedes that of the alburnum. In the radicle the sap appears also evidently to descend* through the cortical vessels,† and in the succulent annual shoot it as evidently passes up through the central

* Phil. Trans. 1805 and 1806.

† I wish it to be understood, that I exclude in these remarks, and in those contained in my former Memoirs, all trees of the palm kind, with the organisation of which I am almost wholly unacquainted.

vessels,* which surround the medulla. In both cases a cellular substance, similar to that which was generated in the preceding experiments, is first formed, and this cellular substance in the same manner subsequently becomes vascular; whence it appears, that the true sap, or blood of the plant, produces similar effects, and passes through similar stages of organisation, when it flows from different sources, and that the power of generating a new bark, properly speaking, belongs neither to the bark nor alburnum, but to a fluid, which pervades alike the vessels of both.

I shall, therefore, not attempt to decide on the merits of the theory of MALPIGHI, or of HALES, respecting the reproduction of the interior bark; but I cannot by any means admit the hypothesis of MALPIGHI and other naturalists, relative to the transmutation of bark into alburnum; and I propose to state my reasons for rejecting that hypothesis, in the next communication I have the honour to address to you.

I am, my dear Sir,

Your much obliged obedient Servant,

T. A. KNIGHT.

Elton, Dec. 18, 1806.

* Phil. Trans. 1805. MIRBEL has called the tubes, which I call the central vessels, the "tissu tubulaire" of the medulla.

VI. *An Investigation of the general Term of an important Series in the inverse Method of finite Differences.* By the Rev. John Brinkley, *D. D. F. R. S. and Andrews Professor of Astronomy in the University of Dublin. Communicated by the Astronomer Royal.*

Read February 26, 1807.

THE theorems relative to finite differences, given by M. LAGRANGE in the Berlin Memoirs for 1772, have much engaged the attention of mathematicians. M. LAPLACE has been particularly successful in his investigations respecting them; yet an important difficulty remained, to endeavour to surmount which is the principal object of this Paper. The theorems alluded to may be thus stated.

Let u represent any function of x. Let $x+h$, $x+2h$, $x+3h$, &c. be successive values of x, and u_1, u_2, u_3 &c. corresponding successive values of u. Let $\Delta^n u$ represent the first term of the nth order of differences of the quantities u, u_1, u_2 &c. And let also $S^n u$ represent the first term of a series of quantities, of which the first term of the nth order of differences is u. Then (e representing the series $1+1+\frac{1}{1.2}+\frac{1}{1.2.3}+$, &c.)

1. $\Delta^n u=(e^{\frac{d}{x}b}-1)^n$ 2. $S^n u=(e^{\frac{d}{x}b}-1)^{-n}$ provided that in the expansion of $(e^{\frac{d}{x}b}-1)^n$, $\frac{d}{x^2}$, $\frac{d}{x^3}$, $\frac{d}{x^4}$ &c. be substituted for

$\left(\frac{d}{x}\right)^n$, $\left(\frac{d}{x}\right)^1$ &c.; and provided that in the expansion of $(e^{\frac{x}{b}}-1)^{-n}$ fl.^{n} ux^n, fl.$^{n-1}$ ux^{n-1}, &c. be substituted for $\left(\frac{d}{x}\right)^{-n}$, $\left(\frac{d}{x}\right)^{-n+1}$ &c. and $\frac{2}{2^n}$, $\frac{3}{3^n}$, &c. be substituted for $\left(\frac{d}{x}\right)^0$, $\left(\frac{d}{x}\right)^1$, &c.

These theorems, which M. LAGRANGE had not demonstrated except by induction, have since been accurately investigated in different ways by M. LAPLACE,* and also by M. ARBOGAST†.

The expanded formula for $S^t u$, or, more accurately speaking, the natural series for $S^t u$ is of the form

$\frac{\alpha}{b^n}$ fl.^{n} $ux^n + \frac{\beta}{b^{n-1}}$ fl.$^{n-1}$ ux^{n-1} . . . . . $\frac{u}{b}$ fl. $ux + ux + \pi \frac{t}{x} lu + \rho \frac{t^2}{x^2} l^2 u$ + &c.

The coefficients α, β, γ, &c. are readily obtained by equations of relation, which were first given by LAGRANGE. But to complete the solution it is obviously necessary to obtain the law of progression, and be able to ascertain any coefficient independent of the preceding ones. This has not hitherto been done, as far as I know, except in the case of $n=1$. M. LAPLACE has given a very ingenious investigation for that case.‡ This has been copied with just encomiums by M. LACROIX,§ who does not mention that any one had accomplished it for any other value of n. In this Paper the general term is given for any value of n, the law of which is remarkably simple. A particular formula, remarkably simple as to its law, is also given for the case of $n=1$, from which that of LAPLACE may be deduced.

* *Mém. Acad. Scien.* 1777 and 1779.

† ARBOGAST *du Calcul des Derivations*, Art. 399, &c.

‡ *Mém. Acad. Scien.* 1777.

§ LACROIX *Traité des Differences et des Series*, Art. 918.

In the case of $n=1$ and $h=1$ the formula for Su is of the form α fl. $ux + \beta u + \gamma \frac{u}{x} + \delta \frac{u}{x^2}$ + &c. and exhibits the sum of a series of which u any function of x is the general term. For which purpose it was first given by EULER in the VIth Vol. *Com. Petropol.* and afterwards demonstrated in the VIIIth Vol. of the same work. It has been differently investigated since by several authors.* But LAPLACE appears to have been the first who gave a general term for the coefficients. EULER seems to have sought it in vain, for he says "Ipsa series "coefficientum α, β, γ &c. ita est comparata ut vix credam "pro eâ terminum generalem posse exhiberi."

As preparatory to the main object of this enquiry it has been thought proper to give investigations of the above two theorems, which will probably be found as simple as any that have appeared. It has also been found necessary, for avoiding very complex formula, to adopt a peculiar notation, which requires some explanation.

Notation.

The first term of the nth order of differences of the series $0^n, 1^n, 2^n$, &c. is denoted by - - - - - $\Delta^n 0^n$

$\frac{x^n}{1.2.3..n}$ is denoted by - - - - - - $\dot{x}^n$

$\frac{\dot{x}^n}{1.2.3..n}$ is denoted by - - - - - - $\frac{n}{\dot{x}}$

$\frac{\Delta^n 0^m}{1.2.3..n.1.2.3..m}$ is denoted by - - - - - $\frac{\Delta^n 0^m}{\dot{n}\ \dot{m}}$

which omission of the denominators cannot produce any inconvenience, because the indices sufficiently point them out whether those indices refer to powers, fluxions, or differences.

* MACLAURIN's Fluxions, Vol. II. p. 672, &c. WARING's Med. Analyt. p. 581, &c.

$\frac{1}{1.2.3..n}$ is denoted by $\underline{1^n}$

$\frac{1}{(1.2.3..n)^m 1.2.3..m}$ is denoted by $\underline{(1^n)^m}$

According to which notation,

$e = 1 + 1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \&c. = 1 + 1 + \underline{1^2} + \underline{1^3} + \&c.$

$e^x = 1 + x + \frac{x^2}{1.2} + \frac{x^3}{1.2.3} + \&c. = 1 + x + \underline{x^2} + \underline{x^3} + \&c.$

If the quantity $\underline{1^p}^t \, \underline{1^q}^v$ &c. or $\frac{1}{(1.2..p)^t (1.2..q)^v 1.2..t.1.2..v \&c.}$ has various values arising from different values of p, q, t, v, &c. then the sum of them all is denoted by $\int \underline{1^p}^t \underline{1^q}^v$ &c.

Theorem I. Let u be a function of x and $x+h$, $x+2h$, $x+nh$ successive values of x. Then $\Delta^n u = (e^{\frac{d}{dx}h} - 1)^n$ if after the expansion of this latter quantity $\frac{d}{dx}$, $\frac{d^2}{dx^2}$, $\frac{d^3}{dx^3}$, &c. be substituted for $\left(\frac{d}{dx}\right)^2$, $\left(\frac{d}{dx}\right)^3$, &c. respectively.

Demonstration. Let the successive values of u be represented by ${}_1u, {}_2u, {}_3u, \ldots {}_{(n-1)}u, {}_nu$. Then by TAYLOR's theorem,

${}_nu = u + \frac{d}{dx}nh + \frac{d^2}{dx^2}\underline{n^2}h^2 + \frac{d^3}{dx^3}\underline{n^3}h^3 + \&c. = u + e^{\frac{d}{dx}nh} - 1$, substituting as above-mentioned. In like manner,

${}_{(n-1)}u = u + e^{\frac{d}{dx}(n-1)h} - 1$, substituting &c.

${}_{(n-2)}u = u + e^{\frac{d}{dx}(n-2)h} - 1$ &c. &c.

${}_2u = u + e^{\frac{d}{dx}h} - 1$ &c. &c.

${}_1u = u + 1 - 1.$

Hence $u-1$ being common to each term, we have by the differential theorem,

$\Delta^n u = e^{\frac{z}{x}nb} - ne^{\frac{z}{x}(n-1)b} + n\left(\frac{n-1}{2}\right)e^{\frac{z}{x}(n-2)b} - \&c. \pm 1 = (e^{\frac{z}{x}b} - 1)^n$ substituting as above-mentioned.

Theorem II. $\Delta^n u = \frac{z}{x^n}h^n + a\frac{z}{x^{n+1}}h^{n+1} + b\frac{z}{x^{n+2}}h^{n+2}$ ---- $M\frac{z}{x^{n+m}}h^{n+m}$ + &c. where M is the coeff. of h^{n+m} in the expansion of $(e^h - 1)^n$ or of h^n in the expansion of $\left(\frac{e^h-1}{h}\right)^n$ or of $(1 + \frac{h}{1.2} + \frac{h^2}{1.2.3} + \&c.)^n$

Dem. By *Theor.* I. $\Delta^n u = (e^{\frac{z}{x}b} - 1)^n$ making the necessary change after expansion. This change does not affect the numerical coefficients, which are evidently the same as those of $(e^h - 1)^n$, and because $(e^h - 1)^n = h^n(1 + \frac{h}{1.2} + \frac{h^2}{1.2.3} + \&c.)^n$ the theorem is manifest.

Theorem III. The coefficient of $\frac{z}{x^{n+m}}h^{n+m} = \Delta^n \underline{o}^{n+m}$

Dem. The successive values of u are

$u, u + \frac{z}{x}h + \frac{z}{x^2}\underline{h^2} + \&c., u + \frac{z}{x}gh + \frac{z}{x^2}g^2\underline{h^2} + \&c., u + \frac{z}{x}g^2h + \frac{z}{x^2}g^3\underline{h^2} + \&c.$ and therefore (the whole nth differences being made up of the nth difference of the parts) we have

$\Delta^n u = \frac{z}{x}h\Delta^n o + \frac{z}{x^2}h^2\Delta^n \underline{o}^2 + \text{---} \frac{z}{x^{n+m}}h^{n+m}\Delta^n \underline{o}^{n+m} + \&c.$

where it is to be observed that whilst $n+m$ is less than n $\Delta^n \underline{o}^{n+m} = 0$, and $\Delta^n \underline{o}^n = 1$, as is well known, and easily appears from *Theorem* II.

Theorems relative to the inverse Method of finite Differences.

Theorem IV. Let u be a function of x and $x+h$, $x+2h$, &c. successive values of x, as before, then

$S^{-n}u=(e^{\frac{d}{x}b}-1)^{-n}$ if after expansion $\left(\frac{d}{x}\right)^{-n}$, $\left(\frac{d}{x}\right)^{-n+1}$ &c. are changed into fl.^{n} ux^{n}, fl.$^{n-1}$ ux^{n-1}, &c. and $\left(\frac{d}{x}\right)^{0}$, $\left(\frac{d}{x}\right)^{1}$, &c. into $\frac{0}{x^{0}}$, $\frac{1}{x^{1}}$, &c.

Demonstration. By *Theorem* II.

$$(1)\text{---}u=A\frac{\overset{n}{(S^{n}u)}}{x^{n}}h^{n}+B\frac{\overset{n+1}{(S^{n}u)}}{x^{n+1}}h^{n+1}+C\frac{\overset{n+2}{(S^{n}u)}}{x^{n+2}}h^{n+2}+\text{ &c.}$$

where A, B, C, &c. are the coefficients of 1, h, h^{2}, &c. in the expansion of $\left(\frac{e^{b}-1}{b}\right)^{n}=(1+\frac{b}{1.2}+\frac{b^{2}}{1.2.3}+\text{, &c.})^{n}$. So that if $v=(1+h+h^{2}+\text{ &c.})^{n}$, making $h=0$, $v=A$, $\frac{dv}{dh}=B$, $\frac{d^{2}v}{dh^{2}}=C$ &c.

$(2)\text{---}$ Let* $\frac{\overset{n}{(S^{n}u)}}{x^{n}}h^{n}=\alpha u+\beta\frac{d}{x^{1}}h+\gamma\frac{d^{2}}{x^{2}}h^{2}+\delta\frac{d^{3}}{x^{3}}h^{3}+\text{ &c.}$ It is evident that this assumption may be made to satisfy the equation (1).

Then, by taking the successive fluxions of equation (2) and substituting in equation (1), we have

* Euler uses a similar assumption in his investigation of the sum of a series from its general term, p. 15, Tom. VIII. *Com. Petropol.*

$$u = \begin{cases} A\alpha u + A\beta \frac{i}{x} h + A\gamma \frac{i^2}{x^2} h^2 + A\delta \frac{i^3}{x^3} h^3 + \&c. \\ \qquad B\alpha \frac{i}{x} h + B\beta \frac{i^2}{x^2} h^2 + B\gamma \frac{i^3}{x^3} h^3 + \&c. \\ \qquad\qquad + C\alpha \frac{i^2}{x^2} h^2 + C\beta \frac{i^3}{x^3} h^3 + \&c. \\ \qquad\qquad\qquad D\alpha \frac{i^3}{x^3} h^3 + \&c. \\ \qquad\qquad\qquad\qquad \&c. \qquad \&c. \end{cases}$$

Hence

$$\begin{aligned} &A\alpha = 1 \\ &A\beta + B\alpha = 0 \\ &A\gamma + B\beta + C\alpha = 0 \qquad (3) \\ &A\delta + B\gamma + C\beta + D\alpha = 0 \\ &\&c. \qquad \&c. \qquad \&c. \end{aligned}$$

In order to obtain the values of α, β, γ, δ, &c. let z represent a function of h which expanded gives $\alpha + \beta h + \gamma h^2 + \delta h^3 + \&c.$ or $z = \alpha + \beta h + \gamma h^2 + \delta h^3 + \&c.$

Then by equations (3) we have, when $h = 0$

(4) $\dot{\dot{\dot{

Let the equation (2) be multiplied by $\frac{x^n}{b^n}$, and the nth fluent taken, and we have

$$S^n u = \frac{\alpha}{b^n} \text{fl.}^n u\dot{x}^n + \frac{\beta}{b^{n-1}} \text{fl.}^{n-1} u\dot{x}^{n-1} + \ldots \frac{\mu}{b} \text{fl.} u\dot{x} + \nu u +$$

$$\pi \frac{\dot{x}}{x} b + \rho \frac{\dot{x}}{x^2} h^2 \text{ \&c.}$$

Also from the above value of x, it is easy to see that the expansion of $\left(\frac{e^{\frac{\dot{x}}{x}h}}{e^{\frac{\dot{x}}{x}h} - 1}\right)^{-n} = \frac{1}{h^n}\left(\frac{h}{e^{\frac{\dot{x}}{x}h} - 1}\right)^n$ gives

in the seventh volume of their Transactions, furnish a general method of reducing any function of x to a series ascending by the powers of x, and that either by assigning at once the coefficient of x^n, or by equations of relation between the coefficients. By the *converse* of that method we are enabled, either from the general coefficient, or from the equations of relation, to arrive at the primitive function. The converse, therefore, applies to the summation of series, to the investigation of the general term of a recurring series, and to several other important purposes. It is evident, that it applies to finding the general term of a recurring series, because from the given scale of relation, the primitive function can be deduced, and from the primitive function, the general coefficient may be determined. The same method extends to the reduction of any function of x, y, z, &c. and therefore the converse to finding the general term of double, triple, &c. recurring series.

I had not considered the converse of the method of reduction of analytical functions afforded by my theorems for finding fluxions *per saltum*, till I had seen M. Arbogast's ingenious work, entitled "Du Calcul des Derivations." As those theorems furnish every thing that is given in the former part of his treatise, and likewise admit of more extensive application; so also the converse of them serve for deducing, with greater facility, every thing respecting recurring series, &c. contained in the same Treatise.

The important uses to be derived from finding fluxions per saltum in the reduction of analytical functions, and from the converse, induced me to draw up a particular work on that subject. Its publication has hitherto been delayed by my unwillingness to offer a fluxional notation different from either that of Newton or Leibnitz, each of which is very inconvenient as far as regards the application of the theorems for finding fluxions *per saltum*.

$\frac{\alpha}{b^n}\left(\frac{x}{x}\right)^{-n} + \frac{\beta}{b^{n-1}}\left(\frac{x}{x}\right)^{-n+1} + \ldots : \frac{\mu}{b}\left(\frac{x}{x}\right)^{-1} + \varkappa u + \varpi\left(\frac{x}{x}\right)h +$
$p\left(\frac{x}{x}\right)^2 h^2 +$ &c.

Whence the truth of the theorem is manifest.

Theorem V. $S^r u = \left(\frac{a}{b^r}\right)$ fl.$^n u x^n + \frac{\beta}{b^{n-1}}$ fl.$^{n-1} u x^{n-1} + \ldots$ &c.

where the numerical coefficient of the $m+1$ term is the coefficient of h^m in the expansion of $\left(1 + \frac{b}{1.2} + \frac{b^2}{1.2.3} + \text{&c.}\right)^{-n}$.

The demonstration of this is contained in that of the last theorem.

Previously to the investigation of the numerical coefficient of the general $m+1$ term, it will be convenient to premise the following lemmas.

Lemma I. Let n represent any affirmative integral number, and m any other affirmative integral number not greater than n. Then

$\int \underline{1^p}^t \underline{1^q}^v \underline{1^r}^w$ &c. $= \underline{\Delta}^n \underline{o}^n$ where p, q, r, &c. : t, v, w, &c. represent any affirmative integral numbers satisfying the equations $t+v+w+$ &c. $= m$

$tp+vq+wr+$ &c. $= n$.

Demonstration.

Let $a^{(1)} = \underline{1}^n$

$a^{(2)} = 1.2 \int \underline{1^p}^t \underline{1^q}^v$ where $t+v=2$ & $tp+vq=n$.

$a^{(3)} = 1.2.3 \int \underline{1^p}^t \underline{1^q}^v \underline{1^r}^w$ where $t+v+w=3$ and $tp+vq+rw=n$.

. . . . . .

$a^{(m)} = 1.2.3..m \int \underline{1^p}^t \underline{1^q}^v \underline{1^r}^w$ &c. where $t+v+w+$ &c. $=m$ and $tp+vq+rw+$ &c. $=n$.

Then we shall find that

$$(1) \quad \underline{m}^n = m\,a^{(1)} + \frac{m(m-1)}{1.2}\,a^{(2)} + \frac{m(m-1)(m-2)}{1.2.3}\,a^{(3)} + \ldots + ma^{(m-1)} + a^{(m)}$$

For taking m quantities α, β, γ, &c. we easily deduce by help of the multinomial theorem,

$$(\underline{\alpha+\beta+\gamma+} \text{ &c.})^n = \underline{\alpha}^n + \underline{\beta}^n + \text{ &c.} + \int \underline{\alpha}^p\,\underline{\beta}^q + \text{ &c.} + \int \underline{\alpha}^p\,\underline{\beta}^q\,\underline{\gamma}^r + \text{ &c.} + \text{ &c.}$$

where $p+q=n$, $p+q+r=n$, &c. &c.

Now if we consider a term $\int \underline{\alpha}^p\,\underline{\beta}^q\,\underline{\gamma}^r$ &c. where k quantities α, β, γ, &c. are concerned, and if $p', p' \ldots$ (t' numbers), $q', q' \ldots$ (v' numbers), $r', r' \ldots$ (w' numbers), denote values of p, q, r, &c. satisfying the equation $p+q+r+$ &c. (k terms) $=n$, we shall see, that the number of the terms, in which these values $p', p', \ldots q', q', \ldots r', r'$, &c. are found, is the number of combination of m things, taking k together into the number of permutations of k things, of which t', v', &c. are the same. This is evident, because in any product $\alpha\beta\gamma$ (k factors), the indices $p', p' \ldots q', q'$, &c. are to be annexed in every possible order. And when the quantities α, β, γ, &c. are each units, each of the quantities is expressed by the same quantity, $1^{t'} \cdot 1^{t'} \ldots 1^{v'} \cdot 1^{v'} \ldots 1^{r'} \cdot 1^{r'}$ &c. therefore the sum of all of them is

$$\frac{m(m-1)\ldots(m-k+1)}{1.2\ldots k} \times \frac{1.2\ldots k}{1.2\ldots t.1\ 2\ldots v\ \&c.} \int \left(\underline{1^p}\right)^{t'} \left(\underline{1^{r'}}\right)^{v'} \text{ &c.}$$ Hence, when α, β, γ, &c. are each unity,

$$\int \underline{\alpha}^p\,\underline{\beta}^q\,\underline{\gamma}^r \text{ &c.} + \text{ &c.} \ (k \text{ quantities}) = \frac{m(m-1)\ldots(m-k+1)}{1.2\ldots k}\,a^{(k)}$$

Hence by substituting for k the numbers 1, 2, 3, &c. we easily obtain the equation (1). From which, substituting for m successively 1, 2, 3, &c. we obtain

$$\underline{1}^n = a^{(1)}$$
$$\underline{2}^n = 2a^{(1)} + a^{(2)}$$
$$\underline{3}^n = 3a^{(1)} + 3a^{(2)} + a^{(3)}$$
. . . . . .
. . . . . .
$$\underline{m}^n = ma^{(1)} + \tfrac{m(m-1)}{1.2}a^{(2)} \ldots\ldots ma^{(m-1)} + a^{(m)}$$

These equations correspond to the equations between the terms of a series of quantities, and the first terms of their respective orders of differences, i. e. $\underline{0}^n$, $\underline{1}^n$, $\underline{2}^n$, &c. correspond to the series, and $a^{(1)}$, $a^{(2)}$, &c. to the respective orders of differences.

Hence we conclude, that $a^{(m)} = \Delta^m \underline{o}^n$, and therefore that $\int \left(\underline{1^p}\right)^t \left(\underline{1^q}\right)$ &c. $= \underline{\Delta}^m \underline{o}^n$.

Lemma II. Every thing being, as in the preceding lemma, except that unity is excluded from the values of p, q, r, &c.

$$\int \left(\underline{1^p}\right)^t \left(\underline{1^q}\right)^v \left(\underline{1^r}\right)^w \text{ &c.} = \underline{\Delta}^m \underline{o}^n - \underline{\Delta}^{m-1} \underline{o}^{n-1} + \tfrac{1}{1.2} \underline{\Delta}^{m-2} \underline{o}^{n-2} - \tfrac{1}{1.2.3} \underline{\Delta}^{m-3} \underline{o}^{n-3} + \ldots \text{(to } m \text{ terms.)}$$

Demonstration. Let $b^{(m)}$, $b^{(m-1)}$, $b^{(m-2)}$, &c, represent $1.2 \ldots m \int \left(\underline{1^p}\right)^t \left(\underline{1^q}\right)^v$ &c.; $1.2 \ldots (m-1) \int \left(\underline{1^p}\right)^t \left(\underline{1^q}\right)^v$ &c.; &c. &c. respectively: these latter quantities being defined, as in the preceding lemma, except that unity is excluded from among the values of p, q, &c.

Then it is easy to see, if $a^{(m)}$, $a^{(m-1)}$, &c. denote as in the preceding lemma,

that $a^{(m)} = b^{(m)} + mb^{(m-1)} + \tfrac{m(m-1)}{1.2} b^{(m-2)} +$ &c.

that $a^{(m-1)} = b^{(m-1)} + (m-1)\, b^{(m-2)} + \frac{(m-1)(m-2)}{1.2} b^{(m-3)} + \&c.$

. . . . . .

. . . . . .

&c. &c.

These equations are the same as the equations between the terms of a series of quantities, and the first terms of their respective orders of differences;

. . . $a^{(m-1)}$, $a^{(m)}$, corresponding to the terms of the series, and . . . $b^{(m-1)}$, $b^{(m)}$, to the first terms of the respective orders of differences.

Whence we conclude, that $b^{(m)} = a^{(m)} - ma^{(m-1)} + \frac{m(m-1)}{1.2} a^{(m-2)} - \&c.$ = (by the preceding lemma) $= \Delta^m \underline{o}^n - m\Delta^{m-1} \underline{o}^{n-1} + \frac{m(m-1)}{1.2} \Delta^{m-2} \underline{o}^{n-2} - \&c.$ and therefore, excluding unity from the values of p, q, &c.

$\int \left(\frac{1^p}{\underline{\ }}\right)^t \left(\frac{1^q}{\underline{\ }}\right)^v \left(\frac{1^r}{\underline{\ }}\right)^w \&c. = \Delta^m \underline{o}^n - \Delta^{m-1} \underline{o}^{n-1} + \frac{1}{1.2} \Delta^{m-2} \underline{o}^{n-2}$

. . . . (*m* terms).

Theorem VI. Let $S^n u = \frac{1}{\dot{x}^n}$ fl. $^n u \dot{x}^n + \frac{c^{(1)}}{\dot{x}^{n-1}}$ fl.$^{n-1} u \dot{x}^{n-1}$ + &c. (*vid. Theor.* V.), then $c^{(m)}$ or the coefficient of the $m+1$ term $= -n\left(\frac{n+2}{1}\right)\left(\frac{n+3}{2}\right) \ldots \left(\frac{n+m}{m-1}\right) \Delta^1 \underline{o}^{m+1} + n(n-1)\left(\frac{n+3}{1}\right)\left(\frac{n+4}{2}\right) \ldots \left(\frac{n+m}{m-2}\right) \Delta^2 \underline{o}^{m+2} - n(n-1)(n-2)\left(\frac{n+4}{1}\right)\left(\frac{n+5}{2}\right) \ldots \left(\frac{n+m}{m-3}\right) \Delta^3 \underline{o}^{m+3} - \&c.$ (to *m* terms and *m* factors in each term).

Demonstration. (By *Theorem* V.) The coefficient of the

$m+1$ term is the coefficient of h^m in the expansion of $\left(1+\frac{h}{1.2}+\frac{h^2}{1.2.3}+\text{&c.}\right)^{-n}=(1+\underline{1}^1 h+\underline{1}^2 h^2+\text{&c.})^{-n}$.

Let $d^{(1)}=\underline{1}^{m+1}$

$d^{(2)}=\int\left(\underline{1^t}\right)^t\left(\underline{1^v}\right)^v$, where $t+v=2$; $pt+qv=m+2$, unity being excluded from the values of p, q, &c.

$d^{(3)}=\int\left(\underline{1^t}\right)^t\left(\underline{1^v}\right)^v\left(\underline{1^w}\right)^w$, where $t+v+w=3$, $pt+qv+rw=m+3$, unity &c.

. . . . . .

Then the coefficient of $h^m=-nd^{(1)}+n(n+1)d^{(2)}-n(n+1)(n+2)d..{}^{(3)}\pm n(n+1)..(n+m-1)d^{(m)}$.

This may easily be deduced from the multinomial Theorem, or more readily from the theorems for finding fluxions *per saltum* in the seventh volume of the Transactions of the Royal Irish Academy.

By *Lemma* II.

$d^{(1)}=\Delta^1\underline{o}^{m+1}$

$d^{(2)}=\Delta^1\underline{o}^{m+2}-\Delta^1\underline{o}^{m+1}$

$d^{(3)}=\Delta^2\underline{o}^{m+3}-\Delta^2\underline{o}^{m+2}+\frac{1}{1.2}\Delta^2\underline{o}^{m+1}$

. . . . . .

$d^{(m)}=\Delta^m\underline{o}^{2m}-\Delta^{m-1}\underline{o}^{2m-1}+\frac{1}{1.2}\Delta^{m-2}\underline{o}^{2m-2}-\text{&c.}$

Whence the coefficient of $h^m=$

n	$-\Delta^1\underline{o}^{m+1}$	$+n(n+1)$	$\Delta^2\underline{o}^{m+2}$	$+n(n+1)\ (n+2)$	$-\Delta^3\underline{o}^{m+3}$ &c.
$n(n+1)$		$n(n+1)\ (n+2)$		$n(n+1)..(n+3)$	
$n(n+1)(n+2)$		$n(n+1)..(n+3)$		$n(n+1)..(n+4)$	
1 2		1.2		1.2	
(m terms)		($m-1$ terms)		($m-2$ terms)	

$= $ (by theorems for figurate numbers)

$$-n\left(\tfrac{n+2}{1}\right)\left(\tfrac{n+3}{2}\right)\ldots\left(\tfrac{n+m}{m-1}\right)\Delta^1 \underline{o}^{m+1}+n(n+1)\left(\tfrac{n+3}{1}\right)\left(\tfrac{n+4}{2}\right)\ldots\left(\tfrac{n+m}{m-2}\right)\Delta^2 \underline{o}^{m+2}-$$

$$n(n+1)(n+2)\left(\tfrac{n+4}{1}\right)\left(\tfrac{n+5}{2}\right)\ldots\left(\tfrac{n+m}{m-3}\right)\Delta^3 \underline{o}^{m+3}+\&c.$$ (to m terms and m factors in each term).

Cor. 1. When $n=1$ the coefficient of h^n $=$

$$=-\frac{m(m+1)}{1.2}\Delta^1 \underline{o}^{m+1}+\frac{(m-1)m(m+1)}{1.2.3}\Delta^2 \underline{o}^{m+2}-\frac{(m-2)(m-1)m(m+1)}{1.2.3.4}$$

$$\Delta^3 \underline{o}^{m+3} \&c.$$

(to m terms).

Cor. 2. The same investigation holds for the coefficient of h^n in the expansion of $\left(1+\frac{b}{1.2}+\frac{b^2}{1.2.3}+\&c.\right)^n$. So that substituting $-n$ for n in the above expression we obtain by *Theorem* III.

$$\Delta^n \underline{o}^{n+m}=+n\left(\tfrac{n-2}{1}\right)\left(\tfrac{n-3}{2}\right)\ldots\left(\tfrac{n-m}{m-1}\right)\Delta^1 \underline{o}^{m+1}\mp n(n-1)\left(\tfrac{n-3}{1}\right)\left(\tfrac{n-4}{2}\right)$$

$$\ldots\left(\tfrac{n-m}{m-2}\right)\Delta^2 \underline{o}^{m+2}\pm\&c.$$

(to m terms and m factors in each term) the upper signs taking place when m is odd, and the under when even.

This corollary furnishes an important theorem, greatly facilitating the computation of differences. It often affords a much more convenient general term for the numerical coefficients in the expression for $\Delta^n u$ than that given in *Theor.* III. *viz.* when m is small compared with n in which case the common method for the computation of $\Delta^n \underline{o}^{n+m}$ would be of little use. As in the following example:

Example. To find the first term of the $n-2$ order of differences of the series x^n, $(x+h)^n$, $(x+2h)^n$, &c.

By *Theorem* III.

$$\Delta^{n-2}x^n = n(n-1)..3x^3h^{n-2}\,\Delta^{n-2}\underline{0}^{n-2} + n(n-1)..2xh^{n-1}\,\Delta^{n-2}\underline{0}^{n-1} + n(n-1)..1h^n\,\Delta^{n-2}\underline{0}^n$$

$$\Delta^{n-2}\underline{0}^{n-2} = 1$$

and, By this *Corollary*

$$\Delta^{n-2}\underline{0}^{n-1} = (n-2)\Delta^1\underline{0}^n = \frac{n-2}{2}$$

$$\Delta^{n-2}\underline{0}^n = -(n-2)(n-4)\Delta^1\underline{0}^3 + (n-2)(n-3)\Delta^2\underline{0}^4 = \frac{(n-2)(3n-5)}{1.2.3.4}$$

Hence

$$\Delta^{n-2}x^n = 1.2..n\left(\frac{x^3}{1.2}h^{n-2} + \frac{n-2}{2}xh^{n-1} + \frac{(n-2)(3n-5)}{1.2.3.4}h^n\right)$$

Scholium.

The case of the above theorem, when $n=1$, has, on account of its importance, been a particular object of investigation among mathematicians. Although the formula in the first corollary is, as to its formation and law of progression, very simple, yet one more simple may be readily obtained by the joint application of a transformation given by M. LAPLACE in his method, and of the first lemma above given; which formula and its investigation are here subjoined. This and the above general formula (n being any number), as well as the formula of LAPLACE (n being 1), do not enable us to compute the successive coefficients so readily as from the equations of relation. But this circumstance, it is imagined, will not render what has been here done less worthy of the notice of mathematicians. Their researches for a general term in the case of

$n=1$, sufficiently shew of what importance the enquiry has been considered.

Theorem. $Su=\frac{1}{b}\text{fl.}\,u\dot{x}-\frac{u}{2}+a\frac{\dot{u}}{\dot{x}}h+b\frac{\overset{3}{\cdot}}{\dot{x}^3}h^3+\ldots P\frac{\overset{m}{u}}{\dot{x}^m}h^m+\&c.$ in which the even powers of h are not found, and P the coeff. of $\frac{\overset{m}{u}}{\dot{x}^m}h^m=\frac{1}{1.2.3\ldots m(2^{m+1}-1)2^m}\left(2^{m-1}\Delta^1 o^m-2^{m-2}\Delta^2 o^m+2^{m-3}\Delta^3 o^m\ldots.+\Delta^m o^m\right)$.

Demonstration. The coefficient of $\frac{\overset{m}{u}}{\dot{x}^m}h^m$, or $c^{(m+1)}$ (*Vid. Theorem* V.) is the coefficient of h^{m+1} in the expansion of $\left(1+\frac{b}{1.2}+\frac{b^2}{1.2.3}+\&c.\right)^{-1}$, or of $\left(\frac{b}{e^b-1}\right)$

1. Now $\frac{b}{e^b-1}+\frac{b}{e^{-b}-1}=-b\ldots\ldots(1)$

Let

$1+Ah+Bh^2\ldots.+Nh^m+Ph^{m+1}+\&c.$ { represent the expansion of $\frac{b}{e^b-1}$, m being any odd number.

Then

$-1+Ah-Bh^2\ldots.+Nh^m-Ph^{m+1}+\&c.$ { will represent the expansion of $\frac{b}{e^{-b}-1}$

Hence by equation (1) we obtain

$A+Ch^2\ldots+Nh^{m-1}+\&c.=-\frac{1}{2}\ldots.(2)$

From whence it follows that

$A=\frac{1}{2}$, $C=0\ldots N=0\ldots.$ Hence when m is any odd number the coefficient of $h^m=0$, and therefore the coefficient of $\frac{\overset{m-1}{u}}{\dot{x}^{m-1}}h^{m-1}$.

$$2.\ \frac{b}{e^b-1}=\frac{\frac{1}{2}b}{e^{\frac{1}{2}b}-1}-\frac{\frac{1}{2}b}{e^{\frac{1}{2}b}+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Let p, q, r, and s represent the coefficients of h^{m+1} in the expansion of $\frac{b}{e^b-1}$, $\frac{\frac{1}{2}b}{e^{\frac{1}{2}b}-1}$, $\frac{b}{e^b+1}$, and $\frac{\frac{1}{2}b}{e^{\frac{1}{2}b}+1}$ respectively: then it is easy to see that $\frac{p}{2^{m+1}}=q$, $\frac{r}{2^{m+1}}=s$ and therefore by equation (3) $p=q-s=\frac{p-r}{2^{m+1}}$, or $p=\frac{r}{1-2^{m+1}}$.

To obtain r the coefficient of h^{m+1} in the expansion of $h\left(2+h+\frac{b^2}{1.2}+\text{&c.}\right)^{-1}$

Let $d^{(1)}=\underline{1}^m$

$d^{(2)}=\int\left(\underline{1}^t\right)\left(\underline{1}^v\right)$ where $t+v=2$ and $pt+qv=m$

$d^{(3)}=\int\left(\underline{1}^t\right)^{\prime}\left(\underline{1}^v\right)^{\prime\prime}\left(\underline{1}^r\right)^{\prime\prime\prime}$ where $t+v+w=3$ and $pt+qv+rw=m$

&c. &c. &c.

Then the coefficient of h^m in the expansion of $\left(2+h+\frac{b^2}{1.2}+\text{&c.}\right)^{-1}=$

$$-\frac{1}{2^1}d^{(1)}+\frac{1.2}{2^2}d^{(2)}-\frac{1.2.3}{2^3}d^{(3)}+\text{&c.}$$

But by *Lemma* I.

$d^{(1)}=\Delta\underline{o}^m$

$1.2\,d^{(2)}=\Delta^2\underline{o}^m$

$1.2.3\,d^{(3)}=\Delta^3\underline{o}^m$

&c. &c.

Hence the coefficient of h^{m+1} in the expansion of $\frac{b}{e^b-1}$ or co-

efficient of $\frac{x^m}{1.2.3..m}h^m$ in the expression for $Su =$

$$\frac{1}{1.2.3..m\left(2^{m+1}-1\right)2^{m+1}}\left(2^{m-1}\Delta^1 o^m - 2^{m-2}\Delta^2 o^m + \ldots\ldots + \Delta^m o^m\right)$$

The same conclusion may be derived somewhat more easily by the assistance of a diverging series, as follows. This investigation, however, is not given as affording the same satisfaction to the mind as the above demonstration,

$$\frac{1}{e^b+1} = 1 - e^b + e^{2b} - \text{\&c.}$$

Hence it is easy to see that the coefficient of h^m
$= -\underline{1}^m + \underline{2}^m - \underline{3}^m + \text{\&c. in infinitum.}$

Also $\frac{1}{e^b+1} = \frac{e^{-b}}{1+e^{-b}} = e^{-b} - e^{-2b} + e^{-3b} - \text{\&c.}$

from which it likewise appears that the coefficient of h^m
$= \pm\underline{1}^m \mp \underline{2}^m \pm \underline{3}^m \mp$ &c., the upper signs taking place when m is even, and the lower when odd. Hence when m is even $-\underline{1}^m + \underline{2}^m - \underline{3}^m +$ &c. $= \underline{1}^m - \underline{2}^m + \underline{3}^m -$ &c. and therefore necessarily $-\underline{1}^m + \underline{2}^m - \underline{3}^m +$ &c. $= 0$. Consequently, when m is even the coefficient of h^m in the expansion of $\frac{1}{e^b+1} = 0$. And generally by the appplication of a well-known theorem,

$$-\underline{1}^m + \underline{2}^m - \underline{3}^m + \text{\&c.} = -\tfrac{1}{2}\Delta^1 \underline{o}^m + \tfrac{1}{2}\Delta^2 \underline{o}^m - \text{\&c.}$$

Whence, &c. &c.

It may be remarked, that in the above theorem the coefficient of h^m can be computed, without using a higher quantity in the series 0^m, 1^m, 2^m, &c. than $\left(\frac{m-1}{2}\right)^m$. For the first terms of the $\frac{m-1}{2}+1$, $\frac{m-1}{2}+2$, &c. orders of differences of the series $\left(-\frac{m-1}{2}\right)^m$, $\left(1-\left(\frac{m-1}{2}\right)\right)^m$, $\left(2-\left(\frac{m-1}{2}\right)\right)^m$, &c. are obtained

without using a higher power than $\left(\frac{m-1}{2}\right)^m$, and thence $\Delta^{\frac{m+1}{2}} o^m \ldots . \Delta^{m-1} o^m$, and it is known, that $\Delta^m o^m = 1 \ldots m$. Thus the computation of the latter half of the terms in $\left(2^{m-1} \Delta^1 o^m - 2^{m-2} \Delta^2 o^m + \&c.\right)$ will be much facilitated.

The computation of $\Delta^{m-1} o^m$, $\Delta^{m-2} o^m$, &c. is also much facilitated by *Cor.* II. *Theorem* V.

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PRESIDENT AND COUNCIL.

METEOROLOGICAL JOURNAL

for January, 1806.

1806	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.	°	Inches.			
Jan. 1	44	8	0	45	53	29,70	96	0,180	N	1	Rain.
	46	2	0	46	55	29,69	91		W	1	Cloudy.
2	36	8	0	36	52	29,70	87	0,040	W	1	Fine.
	42	2	0	42	54	29,81	84		NW	1	Cloudy.
3	31	8	0	34	52	29,83	85		S	1	Cloudy.
	40	2	0	39	53	29,72	96		S	1	Rain.
4	34	8	0	34	51	29,97	94	0,083	W	1	Fair.
	49	2	0	43	53	29,90	92		S	1	Cloudy.
5	41	8	0	41	52	29,80	90	0,105	W	1	Fair.
	46	2	0	46	53	29,91	93		WNW	2	Cloudy.
6	44	8	0	49	52	29,88	95		SW	2	Cloudy.
	53	2	0	53	54	29,85	92		SW	2	Cloudy.
7	40	8	0	40	52	30,13	94	0,043	SW	1	Fair.
	51	2	0	49	55	30,00	94		SSW	1	Rain.
8	48	8	0	48	53	29,60	93	0,070	SSW	2	Cloudy.
	50	2	0	48	55	29,50	80		SSW	2	Fair.
9	37	8	0	38	53	29,68	90		SW	2	Fair.
	49	2	0	44	54	29,72	81		W	2	Fine.
10	40	8	0	42	52	28,73	89	0,170	SW	2	Cloudy. [much wind last night.
	44	2	0	43	55	28,73	78		W	2	Fair.
11	36	8	0	39	52	29,00	87		NW	2	Fine.
	42	2	0	42	55	29,14	78		N	2	Fine.
12	36	8	0	37	51	28,65	92	0,110	E	2	Rain.
	40	2	0	40	53	28,80	92		NE	1	Cloudy.
13	33	8	0	37	51	29,55	87		NW	1	Cloudy.
	40	2	0	40	53	29,71	86		NNW	1	Fine.
14	34	8	0	42	50	29,58	94	0,105	SW	2	Cloudy.
	46	2	0	44	54	29,62	84		WNW	1	Fine.
15	40	8	0	44	51	29,38	87	0,133	WSW	2	Cloudy. [much wind last night.
	51	2	0	47	53	29,52	81		WNW	2	Fair.
16	44	8	0	50	51	29,17	92	0,030	SSW	2	Rain. [much wind last night.
	51	2	0	46	53	28,97	86		S	2	Cloudy.

METEOROLOGICAL JOURNAL
for January, 1806.

1806	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.	°	Inches.			
Jan. 17	37	8	0	38	51	29,48	91	0,095	SW	1	Cloudy.
	46	2	0	44	53	29,69	85		WNW	1	Cloudy.
18	34	8	0	38	51	29,97	86		W	1	Rain.
	50	2	0	46	53	29,84	90		S	1	Rain.
19	45	8	0	45	52	29,59	83	0,280	W	1	Cloudy.
	50	2	0	49	55	29,74	78		WNW	1	Fine.
20	45	8	0	51	53	29,61	93		SW	2	Cloudy.
	54	2	0	54	55	29,56	92		SW	2	Cloudy.
21	42	8	0	48	54	29,74	94		W	1	Fair.
	55	2	0	55	57	29,88	88		W	1	Fair.
22	45	8	0	48	55	30,02	93		SW	1	Cloudy.
	51	2	0	50	57	29,98	91		SSW	1	Cloudy.
23	40	8	0	50	55	29,77	90		SW	2	Cloudy.
	52	2	0	51	57	29,91	81		W	2	Cloudy.
24	45	8	0	50	55	29,47	94	0,075	W	2	Rain.
	50	2	0	48	58	29,54	78		WSW	2	Fair.
25	40	8	0	40	54	29,59	83		WNW	2	Fine.
	46	2	0	46	57	29,64	78		WNW	2	Fine.
26	36	8	0	37	53	29,33	87		E	1	Cloudy.
	42	2	0	42	55	29,26	87		ENE	1	Cloudy.
27	35	8	0	36	52	29,15	90	0,083	E	1	Cloudy.
	41	2	0	41	54	29,05	88		E	1	Cloudy.
28	35	8	0	36	51	29,10	93	0,235	NE	1	Cloudy.
	38	2	0	38	53	29,09	90		NE	1	Cloudy.
29	36	8	0	36	50	29,05	90		NW	1	Cloudy.
	38	2	0	37	53	29,18	87		NW	1	Snow.
30	28	8	0	29	49	29,25	90		W	1	Cloudy.
	35	2	0	35	51	29,15	87		NE	1	Cloudy.
31	30	8	0	30	48	29,31	90		SW	1	Fair.
	38	2	0	38	51	29,37	87		WNW	1	Fair.

METEOROLOGICAL JOURNAL

for February, 1806.

1806	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
Feb. 1	31	7	0	32	48	29.68	90		SW	1	Cloudy.
	39	2	0	39	50	29.83	88		NE	1	Cloudy.
2	32	7	0	33	47	29.92	90		NE	1	Snow.
	38	2	0	38	50	29.92	90		NE	1	Cloudy.
3	30	7	0	33	47	29.79	92		E	1	Cloudy.
	36	2	0	36	48	29.67	87		E	1	Cloudy.
4	32	7	0	33	46	29.52	93		E	1	Snow.
	37	2	0	37	49	29.62	90		E	1	Cloudy.
5	37	7	0	46	47	29.65	96	0,033	S	1	Cloudy.
	53	2	0	52	52	29.64	87		SSE	2	Fair.
6	41	7	0	41	48	29.64	93	0,130	SW	1	Fine.
	50	2	0	49	53	29.73	88		SSW	2	Rain.
7	46	7	0	46	50	29.59	95		S	1	Cloudy.
	53	2	0	52	54	29.62	88		S	1	Fair.
8	42	7	0	42	52	29.68	93		SSW	1	Fine.
	52	2	0	52	54	29.75	88		SW	1	Fair.
9	46	7	0	46	52	29.61	93	0,095	S	2	Cloudy. [Much wind last night.
	53	2	0	52	55	29.61	82		WSW	2	Cloudy.
10	42	7	0	42	53	29.66	92		SSW	1	Fine.
	51	2	0	51	55	29.67	85		S	1	Cloudy.
11	38	7	0	39	53	29.82	90		W	1	Cloudy.
	49	2	0	48	55	29.85	85		S	1	Fair.
12	41	7	0	41	52	29.85	90		NE	1	Cloudy.
	45	2	0	45	55	29.89	84		NW	1	Cloudy.
13	40	7	0	42	52	29.67	93	0,040	SW	1	Cloudy.
	45	2	0	44	55	29.72	82		WNW	1	Cloudy.
14	32	7	0	34	52	29.72	87		S	1	Fine.
	44	2	0	43	53	29.46	88		S	2	Rain.
15	37	7	0	40	51	29.33	92	0,052	E	1	Cloudy.
	44	2	0	44	53	29.47	87		WNW	1	Cloudy.
16	38	7	0	38	52	29.85	92		S	1	Fair.
	47	2	0	47	53	29.82	88		S	1	Cloudy.

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1806	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
Feb. 17	37	7	0	38	51	29.90	92	0,185	SW	1	Cloudy.
	48	2	0	47	54	30,03	83		N	1	Fair.
18	35	7	0	36	51	30,28	93		NE	1	Cloudy.
	44	2	0	44	54	30,27	84		NE	1	Cloudy.
19	39	7	0	39	51	30,21	90		NNE	1	Cloudy.
	44	2	0	44	53	30,13	82		NE	1	Cloudy.
20	40	7	0	40	51	30,04	85		NE	1	Cloudy.
	44	2	0	44	53	30,00	83		E	1	Cloudy.
21	40	7	0	43	51	29.91	90		SE	1	Cloudy.
	50	2	0	50	53	29,86	86		S	2	Cloudy.
22	47	7	0	48	52	29,63	93		S	2	Rain.
	53	2	0	53	54	29,64	76		S	2	Cloudy.
23	40	7	0	41	52	29,77	84		S	2	Cloudy.
	51	2	0	51	54	29,81	86		S	1	Cloudy.
24	46	7	0	46	54	30,12	94		SW	1	Cloudy.
	56	2	0	56	57	30,21	90		SW	1	Fair.
25	49	7	0	49	55	30,24	94		SW	1	Cloudy.
	55	2	0	55	57	30,24	87		SSW	1	Cloudy.
26	48	7	0	48	56	30,11	90		SSW	2	Cloudy.
	53	2	0	50	57	30,12	76		W	2	Fair.
27	38	7	0	43	54	29,98	85		SW	2	Cloudy.
	53	2	0	50	56	29,80	73		W	2	Fair.
28	38	7	0	38	53	30,00	81		WNW	2	Fair.
	44	2	0	44	56	30,14	72		NW	2	Fair.

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1806	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.	°	Inches.	Points.	Str.	
Mar. 1	36	7	0	37	53	30,10	88	0,040	S	1	Rain.
	46	2	0	43	55	30,18	86		NE	1	Cloudy.
2	43	7	0	45	53	30,20	90		W	1	Cloudy.
	52	2	0	51	56	30,20	83		WNW	2	Cloudy.
3	46	7	0	49	54	29,94	83		WNW	2	Cloudy.
	54	2	0	54	56	29,94	80		WNW	2	Cloudy.
4	41	7	0	41	54	30,16	85		N	2	Rain.
	45	2	0	45	56	30,21	87		NE	2	Rain.
5	38	7	0	38	54	30,37	87		NE	1	Hazy.
	45	2	0	45	56	30,38	74		NE	1	Hazy.
6	32	7	0	33	53	30,45	83		NNE	1	Fine.
	40	2	0	40	55	30,42	78		ENE	1	Fair.
7	35	7	0	36	52	30,40	84		NE	1	Cloudy.
	49	2	0	49	55	30,33	80		NE	1	Hazy.
8	41	7	0	42	53	30,25	91		NE	1	Cloudy.
	46	2	0	46	55	30,15	86		NE	1	Cloudy.
9	40	7	0	41	52	29,78	90		SW	1	Cloudy.
	45	2	0	41	55	29,55	88		NW	1	Rain.
10	28	7	0	32	50	29,11	90	0,120	W	2	Snow.
	41	2	0	39	53	28,90	84		NW	2	Sleet.
11	33	7	0	33	50	29,05	88	0,055	WNW	2	Cloudy.
	38	2	0	38	53	29,12	83		NW	2	Cloudy.
12	28	7	0	29	49	29,13	84		NE	1	Cloudy.
	35	2	0	35	51	28,98	82		NE	1	Cloudy.
13	26	7	0	26	47	29,18	85		NW	2	Fine.
	39	2	0	39	51	29,38	77		W	2	Fair.
14	34	7	0	37	48	29,20	90		E	1	Rain.
	50	2	0	49	51	29,03	93		S	2	Rain.
15	33	7	0	33	49	29,50	86	0,100	N	1	Fair.
	43	2	0	43	53	29,57	77		NE	1	Fair.
16	35	7	0	36	49	29,26	92	0,105	E	1	Rain.
	39	2	0	39	52	29,33	93		E	1	Rain.

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1806	Six'a Therm. least and greatest Heat.	Time.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H. M.			Inches.		Inches.	Points.	Str.	
	°		°	°		°				
Mar. 17	38	7 0	39	49	29,38	93	0,098	E	2	Rain.
	39	2 0	39	52	29,30	93		E	2	Rain.
18	39	7 0	40	50	29,31	95	0,190	NE	1	Cloudy.
	48	2 0	47	33	29,35	93		E	1	Rain.
19	38	7 0	39	50	29,40	93	0,040	SW	1	Cloudy.
	47	2 0	47	53	29,35	88		SSW	1	Cloudy.
20	42	7 0	42	51	29,21	94		NE	1	Cloudy.
	48	2 0	47	53	29,30	91		NE	1	Cloudy.
21	40	7 0	41	52	29,72	91		NW	1	Fair.
	52	2 0	52	55	29,83	77		W	1	Fair.
22	44	7 0	46	53	29,85	88		SE	1	Cloudy.
	54	2 0	53	55	29,88	87		S	2	Cloudy.
23	44	7 0	46	53	29,82	93		ESE	1	Cloudy.
	56	2 0	56	56	29,70	83		ESE	2	Fair.
24	48	7 0	49	55	29,68	93	0,105	SW	1	Cloudy.
	54	2 0	53	57	29,70	88		NE	1	Cloudy.
25	48	7 0	48	56	29,60	93	0,110	NE	1	Rain.
	53	2 0	53	58	29,60	90		N	1	Cloudy.
26	46	7 0	46	56	29,71	92	0,365	NE	1	Cloudy.
	51	2 0	51	58	29,77	87		NE	1	Cloudy.
27	46	7 0	46	56	29,85	93		NE	1	Cloudy.
	48	2 0	48	58	29,88	87		E	1	Cloudy.
28	43	7 0	43	56	29,91	87		NE	1	Cloudy.
	45	2 0	45	57	29,90	86		NE	1	Cloudy.
29	41	7 0	41	55	29,90	88		NE	2	Cloudy.
	45	2 0	45	57	29,91	85		NE	1	Cloudy.
30	40	7 0	40	55	29,91	90		NE	1	Cloudy.
	48	2 0	47	57	29,95	80		NE	1	Cloudy.
31	41	7 0	42	54	30,05	87		NE	1	Cloudy.
	48	2 0	48	57	30,10	83		ENE	1	Cloudy.

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1806	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.			Inches.		Inches.	Points.	Str.	
	°			°	°		°				
April 1	36	7	0	38	54	30,30	83		NE	2	Cloudy.
	44	2	0	43	56	30,35	78		NE	2	Cloudy.
2	35	7	0	38	53	30,32	82		NE	2	Cloudy.
	45	2	0	44	55	30,26	77		NE	1	Cloudy.
3	36	7	0	37	51	30,23	84		NE	2	Fair.
	44	2	0	43	54	30,25	77		NE	2	Cloudy.
4	37	7	0	38	52	30,19	85		NE	1	Cloudy.
	43	2	0	42	54	30,12	82		NE	1	Cloudy.
5	35	7	0	38	52	30,01	91	0,080	NE	1	Cloudy.
	49	2	0	48	56	29,97	80		ENE	1	Fair.
6	39	7	0	41	53	29,90	90		ENE	1	Fair.
	55	2	0	55	56	29,88	78		ENE	1	Fair.
7	41	7	0	41	53	29,90	92		ENE	1	Cloudy.
	54	2	0	54	57	29,87	79		E	1	Fair.
8	38	7	0	40	54	29,92	92		NE	1	Cloudy.
	57	2	0	57	57	29,90	87		ENE	1	Fine.
9	42	7	0	42	55	29,85	90		E	1	Cloudy.
	57	2	0	56	59	29,77	84		E	1	Fair.
10	44	7	0	45	55	29,62	92		E	2	Cloudy.
	53	2	0	51	58	29,53	85		E	1	Cloudy.
11	38	7	0	38	55	29,54	88		NE	2	Cloudy.
	44	2	0	43	56	29,57	80		NE	2	Cloudy.
12	35	7	0	36	53	29,66	76		NE	2	Cloudy.
	38	2	0	38	55	29,60	71		NE	2	Cloudy.
13	31	7	0	33	51	29,38	92		NE	2	Snow.
	38	2	0	37	53	29,40	87		NE	2	Sleet.
14	34	7	0	34	50	29,52	93		NE	2	Snow.
	43	2	0	42	52	29,61	85		NNE	2	Cloudy.
15	33	7	0	34	50	29,71	88		NNE	2	Cloudy.
	41	2	0	40	52	29,85	80		NE	2	Cloudy.
16	32	7	0	35	50	30,20	87		NNE	1	Fair.
	50	2	0	49	53	30,31	73		NE	1	Fair.

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1806	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer.	Rain. Inches.	Winds. Points.	Str.	Weather.
Apr. 17	37	7	0	40	50	30,47	84		WSW	1	Cloudy.
	55	2	0	55	54	30,45	76		W	1	Cloudy.
18	45	7	0	47	52	30,40	91		SW	1	Fair.
	58	2	0	57	56	30,36	86		SW	1	Cloudy.
19	51	7	0	51	55	30,32	94	0,165	W	1	Cloudy.
	59	2	0	58	57	30,35	93		S	1	Cloudy.
20	50	7	0	50	55	30,32	93		S	1	Cloudy.
	64	2	0	64	57	30,30	83		NNE	1	Cloudy.
21	50	7	0	51	56	30,30	92		NNE	1	Cloudy.
	59	2	0	58	58	30,27	85		NE	1	Cloudy.
22	49	7	0	50	57	30,23	90		NE	1	Cloudy.
	57	2	0	55	59	30,19	87		NE	1	Rain.
23	44	7	0	45	57	30,20	86		NE	2	Cloudy.
	50	2	0	50	57	30,24	82		NE	2	Cloudy.
24	43	7	0	43	56	30,30	85		NE	2	Cloudy.
	56	2	0	53	58	30,30	81		NE	2	Cloudy.
25	47	7	0	49	57	30,26	85		NNE	2	Cloudy.
	54	2	0	54	58	30,21	80		NNE	2	Cloudy.
26	41	7	0	42	56	30,32	82		NE	2	Cloudy.
	47	2	0	47	57	30,37	76		NE	2	Cloudy.
27	39	7	0	43	55	30,22	81		NW	1	Cloudy.
	55	2	0	55	57	30,04	78		NW	2	Cloudy.
28	43	7	0	42	55	29,79	78		NNE	1	Cloudy.
	51	2	0	49	57	29,80	77		NE	2	Cloudy.
29	38	7	0	43	55	29,74	81		SW	1	Cloudy.
	57	2	0	55	57	29,58	77		W	2	Cloudy.
30	46	7	0	48	56	29,56	79		WNW	2	Fair.
	57	2	0	54	57	29,69	77		N	2	Cloudy.

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1806	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.		Inches.			
May 1	46	7	0	48	56	29.81	87	0.075	SE	1	Cloudy.
2	57	2	0	57	58	29.80	78		S	1	Cloudy.
	50	7	0	51	56	29.61	82		S	2	Cloudy.
	65	2	0	65	59	29.58	70		S	2	Fine.
3	50	7	0	51	58	29.63	90	0.134	NE	1	Rain.
	52	2	0	49	59	29.70	91		NE	1	Rain.
4	43	7	0	46	58	30.00	86	0.150	NE	2	Cloudy.
	54	2	0	54	58	30.08	79		NE	2	Cloudy.
5	43	7	0	46	57	30.19	87		NE	2	Cloudy.
	55	2	0	51	59	30.05	90		ENE	1	Rain.
6	48	7	0	49	58	29.84	91	0.120	NE	1	Cloudy.
	70	2	0	68	61	29.76	81		NE	1	Fine.
7	52	7	0	54	59	29.58	89	0.016	NE	1	Rain.
	72	2	0	71	62	29.56	75		ENE	1	Cloudy.
8	54	7	0	57	61	29.53	85		NE	1	Hazy.
	70	2	0	68	64	29.53	77		E	1	Fine.
9	51	7	0	53	62	29.50	87		NE	1	Cloudy.
	64	2	0	61	63	29.48	83		NE	1	Cloudy.
10	53	7	0	55	62	29.62	88		NE	1	Hazy.
	72	2	0	71	64	29.66	70		S	1	Fine.
11	52	7	0	53	63	29.76	90		NE	1	Cloudy.
	65	2	0	62	63	29.77	83		NNE	1	Fair.
12	49	7	0	51	62	29.81	87	0.105	NNE	1	Cloudy.
	59	2	0	58	62	29.81	84		NE	1	Cloudy.
13	45	7	0	48	61	29.71	87	0.190	ENE	1	Rain.
	55	2	0	54	61	29.71	95		ENE	1	Cloudy.
14	49	7	0	51	60	29.46	92		ENE	1	Rain.
	61	2	0	57	60	29.40	94		E	1	Rain.
15	52	7	0	54	60	29.44	88	0.210	S	2	Cloudy.
	64	2	0	63	61	29.60	73		SSW	2	Cloudy.
16	47	7	0	49	60	29.88	90		S	1	Cloudy.
	66	2	0	64	61	29.95	72		S	1	Fair.

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1806	Six's Therm. least and greatest Heat.	Time.	Therm. Without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H. M.	°	°	Inches.		Inches.	Points.	Str.	
	°					°				
May 17	50	7 0	58	60	30.05	84		ENE	1	Fair.
	65	2 0	65	62	30.06	75		E	1	Fair.
18	51	7 0	54	60	30.26	78		NE	1	Fair.
	69	2 0	69	62	30.32	65		E	1	Fine.
19	48	7 0	53	61	30.43	74		NE	2	Fair.
	67	2 0	67	62	30.37	67		NE	2	Fine.
20	46	7 0	53	61	30.24	80		NE	2	Fine.
	72	2 0	72	64	30.17	68		ENE	2	Fine.
21	52	7 0	55	62	30.15	80		NE	2	Cloudy.
	65	2 0	63	62	30.10	73		NE	2	Cloudy.
22	51	7 0	52	62	30.13	91	0.020	NNE	2	Rain.
	70	2 0	70	64	30.16	72		NE	2	Fair.
23	46	7 0	52	62	30.28	83		NE	2	Fair.
	67	2 0	67	63	30.20	68		NE	2	Fine.
24	49	7 0	55	62	30.04	83		NE	2	Fine.
	71	2 0	71	64	29.98	68		NE	2	Fine.
25	53	7 0	58	63	29.98	89		NE	2	Fine.
	74	2 0	73	67	29.96	69		ENE	2	Fine.
26	55	7 0	60	65	29.90	80		ENE	2	Fine.
	74	2 0	73	67	29.88	76		ESE	2	Fine.
27	55	7 0	59	65	29.95	76		NE	1	Cloudy.
	70	2 0	68	65	30.00	77		ESE	1	Cloudy.
28	54	7 0	56	65	30.06	87		NE	1	Cloudy.
	75	2 0	73	68	30.04	72		NE	1	Hazy.
29	57	7 0	60	66	30.01	87		ENE	1	Cloudy.
	70	2 0	69	66	29.96	81		NE	1	Cloudy.
30	51	7 0	55	65	29.84	83		NE	1	Fair.
	67	2 0	66	66	29.77	75		NE	2	Fair.
31	47	7 0	54	64	29.87	77		NE	1	Cloudy.
	60	2 0	59	64	29.85	74		NE	1	Cloudy.

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1806	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
June 1	49	7	0	54	63	29,96	80		NNE	1	Fair.
	64	2	0	63	63	29,98	76		NE	1	Cloudy.
2	54	7	0	56	63	30,10	83		NE	1	Hazy.
	68	2	0	67	64	30,14	74		NE	1	Cloudy.
3	55	7	0	56	63	30,17	78		S	2	Fair.
	73	2	0	70	64	30,10	72		SSW	1	Cloudy.
4	56	7	0	58	63	29,87	77		SSE	2	Cloudy.
	64	2	0	60	63	29,68	78		S	2	Rain.
5	51	7	0	52	62	29,60	90	0,105	WSW	1	Fair.
	68	2	0	66	63	29,64	83		SSW	1	Cloudy.
6	51	7	0	54	62	29,83	84		W	2	Fair.
	67	2	0	66	63	29,82	76		SW	2	Cloudy.
7	53	7	0	56	62	30,06	85		SW	2	Fine.
	71	2	0	71	64	30,11	73		SW	2	Fair.
8	53	7	0	56	62	30,17	83		SSE	1	Fair.
	75	2	0	75	65	30,15	75		S	2	Fair.
9	57	7	0	58	64	30,21	90		SSW	1	Hazy.
	81	2	0	81	67	30,17	72		S	1	Fine.
10	61	7	0	63	66	30,15	80		E	1	Fine.
	83	2	0	83	70	30,07	70		E	1	Fair.
11	65	7	0	65	69	30,17	83		W	1	Fair.
	75	2	0	75	70	30,28	68		N	1	Fair.
12	54	7	0	60	68	30,53	73		NE	1	Fine.
	72	2	0	72	69	30,48	67		NE	1	Fine.
13	55	7	0	59	67	30,37	53		E	1	Fine.
	77	2	0	76	69	30,27	67		SE	1	Fine.
14	60	7	0	62	68	30,18	73		ENE	1	Fine.
	82	2	0	81	71	30,15	66		NE	1	Fine.
15	63	7	0	63	67	30,25	77		E	1	Fine.
	75	2	0	74	71	30,25	68		E	2	Fine.
16	58	7	0	60	70	30,18	84		SE	1	Hazy.
	78	2	0	77	71	30,16	67		N	1	Fine.

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1806	Six's Therm. mean and greatest Heat.	Time.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		M.	M.					Points.	Str.	
	°		°	°	Inches.		Inches.			
June 17	54	7	58	68	30,25	75		NE	1	Cloudy.
	70	2	70	70	30,17	68		NE	1	Fair.
18	50	7	55	68	30,21	73		SW	1	Fine.
	69	2	67	69	30,26	67		NNE	1	Fine.
19	48	7	56	67	30,40	77		NE	1	Fair.
	71	2	71	68	30,40	65		NE	1	Fine.
20	54	7	58	68	30,44	70		S	1	Fine.
	81	2	80	70	30,33	66		W	1	Fine.
21	55	7	59	68	30,20	75		NW	1	Fine.
	72	2	70	68	30,14	72		NW	1	Fair.
22	56	7	58	68	30,07	85		NW	1	Cloudy.
	70	2	69	68	29,95	72		WNW	1	Fair.
23	50	7	56	66	30,02	75		NE	2	Fair.
	62	2	60	66	30,08	65		NE	2	Fair.
24	46	7	52	64	30,14	75		N	1	Fine.
	68	2	66	65	30,09	66		NW	1	Fine.
25	53	7	58	64	30,00	80		W	1	Cloudy.
	68	2	67	65	30,06	72		W	1	Cloudy.
26	55	7	57	64	29,86	80		S	1	Cloudy.
	61	2	60	64	29,79	86		S	1	Rain.
27	56	7	56	64	29,70	94	0,340	NE	1	Rain.
	62	2	59	64	29,85	87		NE	1	Cloudy.
28	48	7	52	63	30,10	81	0,060	NE	1	Fine.
	64	2	63	63	30,12	68		NE	1	Fair.
29	50	7	52	62	30,22	80		E	1	Fine.
	72	2	70	64	30,18	67		S	2	Fair.
30	52	7	60	63	30,03	87		SSW	2	Cloudy.
	65	2	65	64	29,91	92		SW	1	Rain.

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1806		Six's Therm. least and greatest Hour.	Time. H.	M.	Therm. without. °	Therm. within. °	Baron. Inches.	Hygrometer. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
July	1	52	7	0	57	64	29.97	80	0.063	W	2	Cloudy.
		66	2	0	66	64	30.05	73		WNW	1	Cloudy.
	2	53	7	0	56	63	30.17	82		N	1	Cloudy.
		69	2	0	68	64	30.21	72		N	1	Cloudy.
	3	61	7	0	63	64	30.11	80		W	1	Cloudy.
		74	2	0	73	65	30.04	72		WNW	1	Cloudy.
	4	61	7	0	61	64	29.92	88		E	1	Rain.
		68	2	0	66	65	29.92	81		E	1	Cloudy.
	5	53	7	0	57	64	29.95	84	0.030	E	1	Fine.
		72	2	0	71	66	29.92	72		E	1	Fair.
	6	59	7	0	60	64	29.85	85		NE	1	Cloudy.
		62	2	0	61	65	29.85	84		NNE	1	Cloudy.
	7	54	7	0	55	64	29.68	82	0.087	WSW	1	Cloudy.
		67	2	0	63	64	29.55	84		S	2	Cloudy.
	8	54	7	0	56	64	29.64	85	0.095	SW	1	Fine.
		72	2	0	69	65	29.66	78		S	2	Cloudy.
	9	62	7	0	63	65	29.83	91		S	1	Cloudy.
		74	2	0	73	66	29.96	78		S	1	Cloudy.
	10	58	7	0	61	66	30.15	87		SW	1	Hazy.
		81	2	0	81	68	30.13	72		S	1	Fair.
	11	67	7	0	72	68	29.97	81		E	2	Fair.
		73	2	0	68	68	29.82	85		SW	1	Cloudy.
	12	57	7	0	59	69	30.03	84	0.643	SW	1	Rain.
		74	2	0	74	69	30.05	67		WNW	1	Fair.
	13	62	7	0	62	68	29.91	83		SW	2	Cloudy.
		77	2	0	76	69	29.90	66		SW	2	Rain.
	14	64	7	0	65	69	29.74	85	0.102	SSW	2	Cloudy.
		71	2	0	67	69	29.68	85		SW	2	Rain.
	15	55	7	0	57	67	29.85	87	0.625	SW	1	Fine.
		72	2	0	70	68	29.82	73		S	2	Fair.
	16	56	7	0	57	67	29.65	83		SW	2	Fine.
		70	2	0	69	68	29.65	73		WSW	2	Cloudy.

this fore-noon much lightning & thunder.

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1806	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.	°	Inches.			
July 17	53	7	0	57	66	29,70	85		SW	2	Fine.
	68	2	0	68	67	29,68	78		SW	2	Rain.
18	53	7	0	57	66	29,75	84	0,210	S	2	Cloudy. [Lightning, with thunder
	67	2	0	62	66	29,76	82		SW	1	Rain.
19	54	7	0	56	65	29,78	86	0,392	NW	1	Cloudy.
	67	2	0	65	68	29,86	77		N	1	Fair.
20	53	7	0	56	65	29,96	83	0,040	WSW	1	Cloudy.
	70	2	0	67	64	29,95	78		W	1	Cloudy.
21	56	7	0	57	65	29,85	84		SW	1	Fair.
	70	2	0	66	65	29,81	74		SSW	1	Cloudy.
22	52	7	0	55	65	29,82	87	0,092	SW	1	Foggy.
	71	2	0	71	65	29,78	72		E	1	Fair.
23	57	7	0	57	64	29,54	87	0,020	E	1	Rain.
	69	2	0	68	65	29,51	90		E	1	Rain.
24	58	7	0	61	65	29,60	100	0,565	NE	2	Cloudy.
	68	2	0	63	65	29,62	87		SE	1	Rain.
25	58	7	0	60	64	29,61	100	1,433	NE	3	Cloudy. [much lightning, with thunder & heavy rain.
	68	2	0	67	65	29,63	78		NNE	1	Cloudy.
26	55	7	0	56	65	29,60	86		W	1	Fine.
	72	2	0	70	66	29,59	70		W	1	Fair.
27	55	7	0	58	65	29,67	88		SW	1	Cloudy.
	72	2	0	70	66	29,63	71		SSW	1	Cloudy.
28	61	7	0	59	65	29,59	91		ESE	1	Hazy.
	71	2	0	70	66	29,59	78		SE	1	Cloudy.
29	58	7	0	59	66	29,65	90		E	1	Fine.
	73	2	0	73	67	29,67	75		NE	1	Fine.
30	59	7	0	61	66	29,83	85	0,250	NE	1	Cloudy.
	75	2	0	69	67	29,83	80		SW	2	Rain.
31	60	7	0	61	66	29,78	91	0,252	S	2	Cloudy.
	74	2	0	73	68	29,72	70		SW	2	Fair.

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1806	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
Aug. 1	57	7	0	60	67	29,80	87		SSW	2	Cloudy.
	69	2	0	66	67	29,69	80		S	2	Cloudy.
2	57	7	0	58	66	29,55	83	0,042	SW	2	Fine.
	70	2	0	69	67	29,54	73		WSW	2	Fair.
3	56	7	0	58	66	29,78	83	0,200	WSW	1	Cloudy.
	71	2	0	70	67	29,82	70		WSW	1	Fair.
4	54	7	0	58	66	29,95	83		W	1	Cloudy.
	71	2	0	68	67	30,03	73		NNE	2	Fair.
5	55	7	0	57	66	30,14	80		WSW	1	Fair.
	72	2	0	71	67	30,06	68		WSW	1	Cloudy.
6	56	7	0	57	66	29,98	88		W	1	Cloudy.
	72	2	0	70	67	30,00	68		W	1	Cloudy.
7	60	7	0	62	66	29,98	90		WSW	1	Cloudy.
	76	2	0	75	67	29,97	74		W	1	Cloudy.
8	60	7	0	62	67	29,96	86		SW	1	Fine.
	77	2	0	76	68	29,94	72		SSE	1	Cloudy.
9	61	7	0	64	67	29,88	87		E	1	Fine.
	80	2	0	79	70	29,84	71		ESE	1	Fair.
10	58	7	0	59	67	29,94	83		WSW	1	Fine.
	76	2	0	75	70	29,97	70		NW	1	Fine.
11	58	7	0	61	69	30,00	77		NE	1	Fair.
	78	2	0	74	70	29,98	70		NE	1	Cloudy.
12	61	7	0	62	68	29,94	82		SW	1	Fair.
	76	2	0	74	70	29,90	70		WSW	1	Fair.
13	55	7	0	57	68	29,81	86	0,300	SW	1	Rain.
	68	2	0	67	68	29,68	83		SW	1	Rain.
14	54	7	0	56	67	29,78	83	0,660	WNW	1	Fair.
	70	2	0	67	68	29,77	71		WNW	1	Fair.
15	53	7	0	55	66	29,91	82	0,175	WSW	1	Fine.
	69	2	0	68	67	29,98	72		WSW	1	Fair.
16	55	7	0	57	66	30,17	84		SW	1	Fine.
	72	2	0	71	67	30,18	70		WNW	1	Fair.

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1806	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.			Inches.		Inches.	Points.	Str.	
	°			°	°		°				
Aug. 17	58	7	0	60	67	30,20	80		SW	1	Cloudy.
	76	2	0	75	67	30,17	68		SSW	1	Fair.
18	57	7	0	60	67	30,12	82		SSW	1	Fine.
	75	2	0	75	69	30,09	70		E	1	Fine.
19	57	7	0	62	68	30,00	87		NE	1	Cloudy.
	73	2	0	73	69	29,92	74		E	1	Fair.
20	62	7	0	64	68	29,78	91	0,035	NE	1	Cloudy.
	73	2	0	72	69	29,74	85		S	1	Rain.
21	61	7	0	63	68	29,67	88	0,280	S	2	Cloudy.
	71	2	0	68	69	29,66	78		SE	2	Cloudy.
22	61	7	0	59	68	29,75	87		SW	2	Fine.
	76	2	0	70	69	29,83	70		SW	2	Fine.
23	60	7	0	63	69	29,93	86		SW	2	Cloudy.
	74	2	0	73	70	29,93	72		S	2	Fine.
24	57	7	0	58	69	30,07	84		SW	1	Fine.
	72	2	0	70	69	30,09	70		W	1	Fair.
25	53	7	0	58	68	30,18	85		SW	1	Fine.
	71	2	0	71	69	30,06	69		S	1	Fair.
26	60	7	0	62	67	29,64	81		S	2	Cloudy.
	63	2	0	63	67	29,44	86		S	2	Rain.
27	55	7	0	55	66	29,37	86	0,160	S	2	Rain.
	68	2	0	67	67	29,56	73		SW	2	Fair.
28	58	7	0	56	66	29,68	83	0,055	S	2	Cloudy.
	68	2	0	67	66	29,57	73		SSE	2	Cloudy.
29	59	7	0	62	65	29,28	84		E	1	Cloudy.
	67	2	0	65	66	29,25	84		E	1	Rain.
30	53	7	0	56	62	29,63	84	0,180	SW	1	Cloudy.
	66	2	0	66	65	29,68	78		S	1	Fair.
31	51	7	0	54	64	29,77	86		E	1	Fine.
	71	2	0	70	65	29,80	72		S	1	Fine.

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1806	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°						°				
Sep. 1	54	7	0	55	64	29,84	87		SSW	1	Fair.
	72	2	0	71	66	29,91	73		WSW	1	Fair.
2	57	7	0	59	64	30,06	88		WSW	1	Cloudy.
	71	2	0	70	66	30,11	73		WNW	1	Cloudy.
3	59	7	0	61	65	30,06	85		SE	1	Cloudy.
	71	2	0	70	68	29,98	72		SSW	1	Fair.
4	60	7	0	60	65	29,82	90		S	1	Cloudy.
	68	2	0	68	66	29,71	78		S	1	Cloudy.
5	52	7	0	52	64	29,82	86	0,245	SW	1	Fine. [much Lightning last night.
	68	2	0	68	67	29,97	72		W	1	Fair.
6	53	7	0	56	64	30,05	84		NE	1	Hazy.
	71	2	0	71	67	30,00	71		E	1	Fine.
7	59	7	0	61	66	29,93	86		SE	1	Cloudy.
	73	2	0	73	68	29,95	76		S	2	Fair.
8	58	7	0	58	67	29,95	87		S	1	Fine.
	71	2	0	71	68	30,00	70		SW	1	Fine.
9	56	7	0	56	67	29,83	86		NE	1	Rain.
	57	2	0	55	65	29,53	98		SW	1	Rain.
10	50	7	0	52	64	29,74	89	0,950	WSW	1	Fair.
	66	2	0	65	65	29,76	75		SW	1	Fair.
11	51	7	0	52	63	29,64	87	0,120	W	1	Cloudy.
	62	2	0	62	64	29,71	73		NW	1	Fair.
12	48	7	0	49	62	29,91	83		WNW	1	Fine.
	62	2	0	61	62	30,00	76		NW	1	Fair.
13	50	7	0	51	61	30,12	77		N	2	Cloudy.
	62	2	0	61	62	30,14	76		N	1	Fair.
14	49	7	0	51	61	30,04	92		NE	1	Cloudy.
	65	2	0	64	62	29,91	82		NE	1	Fair.
15	54	7	0	54	61	29,84	88		NE	2	Cloudy.
	58	2	0	56	60	29,95	81		NNE	1	Cloudy.
16	46	7	0	48	59	30,12	84	0,130	NE	2	Fine.
	61	2	0	61	61	30,17	70		N	1	Fine.

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		H. M.	°	°	Inches.		Inches.	Points.	Str.	
	°					°				
Sep. 17	50	7 0	50	59	30,25	86		NW	1	Fair.
	65	2 0	63	60	30,27	71		N	1	Cloudy.
18	49	7 0	51	59	30,27	82		W	1	Cloudy.
	66	2 0	66	61	30,24	77		SW	1	Fair.
19	58	7 0	59	60	30,26	95		SSW	1	Rain.
	68	2 0	68	62	30,24	80		SW	1	Fair.
20	59	7 0	60	62	30,21	90		SW	1	Cloudy.
	70	2 0	69	63	30,20	77		WSW	1	Fair.
21	56	7 0	57	62	30,20	91		S	2	Cloudy.
	69	2 0	69	63	30,16	78		SW	1	Fair.
22	50	7 0	51	62	30,02	90		SW	1	Fair.
	71	2 0	71	65	29,96	78		S	1	Fine.
23	55	7 0	55	63	29,96	91		SW	1	Cloudy.
	66	2 0	66	64	29,97	81		S	2	Cloudy.
24	53	7 0	53	63	30,05	90		NNE	2	Cloudy.
	67	2 0	59	62	30,11	80		NE	1	Cloudy.
25	46	7 0	48	61	30,24	85		NE	1	Fine.
	61	2 0	61	62	30,26	78		NE	1	Fine.
26	44	7 0	44	60	30,27	87		SW	1	Fine.
	65	2 0	64	62	30,22	74		SW	1	Fine.
27	53	7 0	55	60	30,19	95		SW	1	Cloudy.
	67	2 0	66	62	30,17	80		S	1	Fine.
28	51	7 0	52	61	30,12	90		E	1	Cloudy.
	64	2 0	63	62	30,07	82		E	1	Cloudy.
29	58	7 0	58	61	29,86	96		NE	1	Cloudy.
	68	2 0	64	62	29,80	95		E	1	Rain.
30	51	7 0	51	61	29,90	93	0,475	SW	1	Fair.
	63	2 0	63	63	29,94	78		SSW	1	Fair.

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1806	Sun's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
Oct. 1	47	7	0	47	59	30,02	90		S	1	Foggy.
	60	2	0	60	60	30,02	77		S	1	Fair.
2	47	7	0	49	59	29.98	91		SW	1	Fair.
	62	2	0	62	61	29.93	81		S	1	Fair.
3	56	7	0	58	60	29,70	97	0,070	S	1	Rain.
	63	2	0	60	61	29.57	90		S	2	Rain.
4	54	7	0	54	60	29,61	95	0,100	E	1	Cloudy.
	63	2	0	63	61	29.75	83		W	1	Cloudy.
5	52	7	0	52	60	30,00	88		W	1	Fair.
	64	2	0	63	61	30,03	80		WSW	1	Fair.
6	57	7	0	57	60	30,09	88		WSW	1	Cloudy.
	65	2	0	64	62	30,16	73		W	1	Fair,
7	48	7	0	49	60	30,33	87		SSW	1	Cloudy.
	59	2	0	59	60	30,33	82		WNW	1	Cloudy.
8	56	7	0	56	60	30,29	90		SW	1	Cloudy.
	61	2	0	60	61	30,27	70		WNW	1	Cloudy.
9	54	7	0	55	60	30,23	80		NE	1	Cloudy.
	63	2	0	63	61	30,18	74		E	1	Cloudy.
10	52	7	0	54	60	30,10	92		NE	1	Cloudy.
	60	2	0	60	61	30,07	78		E	1	Cloudy.
11	53	7	0	54	60	30,02	82		ENE	1	Cloudy.
	60	2	0	59	62	30,02	77		E	1	Fine.
12	48	7	0	49	60	30,02	86		E	1	Fine.
	59	2	0	58	62	30,01	76		E	1	Fine.
13	45	7	0	46	60	29.92	88		NE	1	Fine.
	59	2	0	59	62	29.84	80		E	1	Fine.
14	53	7	0	55	60	29.67	95		E	1	Cloudy.
	62	2	0	62	62	29.69	84		E	1	Cloudy.
15	50	7	0	50	60	29.74	95	0,088	SW	1	Cloudy.
	60	2	0	60	61	29.75	84		SSW	1	Fine.
16	52	7	0	53	60	29.77	92		ESE	1	Cloudy.
	56	2	0	56	60	29.74	84		ESE	1	Cloudy.

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		H. M.			Inches.		Inches.	Points.	Str.	
	°		°	°		°				
Oct. 17	46	7 0	47	58	29,83	82		E	1	Fine.
	54	2 0	54	60	29,80	76		E	1	Fine.
18	43	7 0	43	58	29,58	87		NE	1	Fair.
	54	2 0	53	59	29,52	76		NE	1	Fair.
19	44	7 0	48	55	29,51	87		NE	2	Fair.
	57	2 0	55	58	29,57	78		NE	2	Fair.
20	42	7 0	43	57	29,67	86		NE	1	Cloudy.
	53	2 0	53	58	29,60	81		E	1	Cloudy.
21	45	7 0	51	57	29,32	87		SE	2	Cloudy.
	56	2 0	55	58	29,05	92		S	1	Rain.
22	47	7 0	51	57	28,98	93	0,220	S	1	Cloudy.
	51	2 0	51	58	28,66	95		NE	1	Rain.
23	39	7 0	40	53	29,59	84	0,315	WNW	2	Fine.
	46	2 0	46	57	29,87	74		WNW	2	Fine.
24	35	7 0	38	54	30,05	82		W	1	Cloudy.
	49	2 0	49	57	30,17	75		W	1	Fine.
25	42	7 0	43	54	30,27	82		S	1	Cloudy.
	51	2 0	51	58	30,15	78		S	2	Cloudy.
26	50	7 0	52	55	30,00	87		S	2	Fine.
	64	2 0	64	61	30,01	80		S	1	Fine.
27	51	7 0	52	57	30,00	93		S	1	Fair.
	62	2 0	61	60	29,94	80		S	1	Fair.
28	51	7 0	53	58	29,79	92		SSE	1	Fair.
	62	2 0	62	60	29,85	87		S	1	Cloudy.
29	47	7 0	47	58	30,18	92		SW	1	Fair.
	56	2 0	55	59	30,23	91		WSW	1	Fair.
30	46	7 0	47	58	30,18	94		SE	1	Foggy.
	55	2 0	55	60	30,11	88		E	1	Fine.
31	47	7 0	49	58	29,90	92		E	1	Cloudy.
	53	2 0	53	58	29,82	88		E	1	Cloudy.

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1806	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
Nov. 1	51	7	0	52	58	29,70	93	0,030	E	1	Rain.
	60	2	0	60	59	29,60	91		S	1	Fair.
2	51	7	0	53	59	29,34	90		SSE	2	Fair.
	56	2	0	56	59	29,20	90		S	2	Rain.
3	49	7	0	50	58	29,07	90	0,130	S	2	Rain.
	55	2	0	55	59	29,06	84		S	2	Cloudy.
4	50	7	0	50	58	29,14	94	0,300	NE	1	Rain.
	53	2	0	52	59	29,18	92		N	1	Cloudy.
5	48	7	0	48	58	29,40	93	0,445	W	1	Cloudy.
	53	2	0	52	59	29,48	80		W	1	Fair.
6	38	7	0	39	57	29,66	88		W	1	Fair.
	49	2	0	49	59	29,83	80		NW	1	Fine.
7	36	7	0	36	56	30,17	87		W	1	Fair.
	50	2	0	50	59	30,24	84		W	1	Fine.
8	45	7	0	47	57	30,30	90		S	1	Fair.
	56	2	0	56	57	30,30	82		S	1	Cloudy.
9	46	7	0	46	56	30,28	91		SSE	1	Fair.
	53	2	0	53	57	30,22	80		S	1	Cloudy.
10	41	7	0	42	55	30,07	88		SE	1	Cloudy.
	50	2	0	49	55	30,10	88		E	1	Foggy.
11	47	7	0	48	55	30,21	93		E	1	Cloudy.
	51	2	0	51	57	30,18	85		E	1	Cloudy.
12	38	7	0	38	55	30,07	90		E	1	Foggy.
	47	2	0	46	55	30,04	91		W	1	Cloudy.
13	44	7	0	46	54	30,06	93		SW	1	Cloudy.
	56	2	0	54	56	30,17	87		SW	1	Cloudy.
14	47	7	0	51	56	30,23	92		SW	1	Cloudy.
	53	2	0	53	58	30,15	93		SW	1	Cloudy.
15	51	7	0	51	56	29,93	93		SW	1	Cloudy.
	54	2	0	54	57	29,85	93		SW	1	Cloudy.
16	43	7	0	43	56	29,78	90	0,020	SW	1	Fair.
	51	2	0	51	57	29,75	82		SW	1	Cloudy.

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1806	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.		Inches.			
Nov. 17	43	7	0	46	56	29,94	90		WSW	1	Cloudy.
	54	2	0	53	57	30,00	84		SW	1	Cloudy.
18	48	7	0	52	56	29,88	93		S	2	Rain.
	56	2	0	56	57	29,80	91		S	2	Cloudy.
19	51	7	0	51	56	29,62	90	0,042	SW	2	Cloudy.
	56	2	0	52	57	29,56	86		SW	2	Cloudy.
20	41	7	0	41	54	29,35	85	0,560	WSW	2	Fine.
	49	2	0	48	56	29,44	78		W	2	Fine.
21	42	7	0	47	54	29,13	87		SSW	2	Rain.
	48	2	0	47	56	29,05	81		SW	2	Cloudy.
22	37	7	0	37	53	29,35	88		SW	1	Fine.
	47	2	0	47	57	29,53	80		W	2	Fine.
23	36	7	0	39	53	29,72	88		SW	1	Cloudy.
	53	2	0	49	54	29,51	93		S	2	Rain.
24	47	7	0	50	53	29,85	95	0,093	SW	1	Cloudy.
	56	2	0	56	56	29,92	95		SW	1	Cloudy.
25	43	7	0	46	53	29,71	95		SSW	2	Cloudy.
	58	2	0	56	57	29,71	93		SW	1	Cloudy.
26	51	7	0	51	54	29,38	90	0,280	SW	2	Rain.
	52	2	0	52	57	29,38	91		SW	2	Rain.
27	46	7	0	46	56	29,90	90	0,286	WNW	1	Fair.
	50	2	0	50	57	30,07	87		WNW	1	Cloudy.
28	49	7	0	56	57	30,00	97	0,305	S	1	Rain.
	59	2	0	58	58	30,00	95		S	2	Cloudy.
29	56	7	0	56	57	29,77	95	0,040	SW	2	Cloudy.
	59	2	0	58	59	29,60	86		SW	2	Cloudy.
30	41	7	0	42	55	29,80	87	0,020	SW	2	Fine.
	48	2	0	47	57	29,79	83		WNW	2	Fair.

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1806.	Six's Therm. least and greatest Heat.	Time. H.	M	Therm. without.	Therm. within.	Barom. Inches.	Hygrometer.	Rain. Inches.	Winds. Points.	Str.	Weather.
	°			°	°		°				
Dec. 1	42	8	0	46	55	29,15	90	0,070	SW	2	Rain.
	52	2	0	52	57	29,05	83		SW	1	Cloudy.
2	42	8	0	42	54	28,72	90	0,555	WSW	2	Rain.
	44	2	0	43	56	28,82	79		WNW	2	Cloudy.
3	40	8	0	42	54	29,58	88		WNW	1	Cloudy.
	46	2	0	46	56	29,70	83		NW	1	Cloudy.
4	41	8	0	41	53	29,75	91		N	1	Foggy.
	47	2	0	46	56	29,75	81		NW	1	Cloudy.
5	46	8	0	51	54	29,52	97		SW	2	Cloudy.
	54	2	0	54	56	29,46	83		SW	2	Cloudy.
6	52	8	0	53	55	29,33	90		S	2	Cloudy.
	54	2	0	51	57	29,37	83		SSW	2	Cloudy.
7	40	8	0	40	54	29,33	85		S	2	Fine.
	49	2	0	49	55	29,20	90		S	2	Rain.
8	44	8	0	44	53	29,07	88	0,105	S	1	Cloudy.
	46	2	0	46	54	29,12	86		E	1	Fair.
9	42	8	0	44	53	29,37	88	0,053	WNW	1	Cloudy.
	47	2	0	47	55	29,46	87		NW	1	Cloudy.
10	40	8	0	42	52	29,38	90	0,058	W	1	Cloudy.
	45	2	0	44	53	29,38	83		NW	1	Cloudy.
11	43	8	0	45	52	29,37	91		WSW	1	Cloudy.
	51	2	0	49	55	29,38	86		SSW	1	Cloudy.
12	43	8	0	43	53	29,32	85		SW	1	Fine.
	50	2	0	50	56	29,43	82		SSW	1	Fair.
13	48	8	0	56	53	29,10	92	0,145	S	2	Rain.
	57	2	0	54	57	29,08	90		S	2	Rain.
14	48	8	0	48	54	29,45	85	0,255	S	1	Cloudy.
	50	2	0	48	56	29,40	90		S	1	Rain.
15	45	8	0	45	53	29,63	91	0,610	SSW	1	Rain.
	47	2	0	46	56	29,75	85		SW	1	Fine.
16	42	8	0	53	54	29,67	93	0,095	SW	2	Rain.
	56	2	0	56	57	29,65	90		SSW	2	Cloudy.

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1806	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.		Inches.			
Dec. 17	54	8	0	54	55	29,67	92	0,020	SSW	2	Cloudy.
	56	2	0	56	58	29,68	90		SSW	2	Cloudy.
18	50	8	0	50	56	29,70	91	0,130	SSW	1	Rain.
	52	2	0	50	58	29,75	88		S	1	Rain.
19	48	8	0	49	57	29,61	88	0,090	SE	1	Rain.
	53	2	0	51	58	29,62	85		W	1	Fine.
20	45	8	0	49	56	29,54	88	0,020	SE	2	Fine.
	53	2	0	52	58	29,34	84		S	2	Fair.
21	40	8	0	42	56	29,38	85	0,140	ESE	1	Cloudy.
	47	2	0	45	58	29,32	88		NE	2	Rain.
22	39	8	0	50	55	29,52	92	0,270	S	2	Rain.
	57	2	0	56	57	29,54	88		S	2	Cloudy.
23	52	8	0	55	57	29,87	95		SW	1	Cloudy.
	57	2	0	56	58	29,96	93		SSW	1	Rain.
24	55	8	0	57	57	30,02	90		SSW	2	Cloudy.
	57	2	0	51	59	30,20	85		N	2	Fair.
25	43	8	0	53	56	30,23	97		SSW	2	Rain.
	55	2	0	55	59	30,10	88		SSW	2	Cloudy.
26	42	8	0	43	56	30,24	90	0,025	WSW	1	Fine.
	47	2	0	47	58	30,28	82		WSW	1	Fine.
27	42	8	0	45	55	30,20	92		SW	1	Fair.
	52	2	0	52	57	30,13	88		SW	1	Cloudy.
28	47	8	0	49	56	29,93	92		SW	1	Cloudy.
	53	2	0	53	58	29,84	85		SSW	1	Fine.
29	48	8	0	49	56	29,60	88		SSE	2	Cloudy.
	51	2	0	51	56	29,47	90		S	2	Rain.
30	43	8	0	47	55	29,57	92	0,016	SSW	1	Rain.
	54	2	0	54	56	29,59	90		SSW	1	Cloudy.
31	43	8	0	43	55	30,05	85	0,060	NNE	1	Cloudy.
	44	2	0	43	55	30,20	83		NE	1	Cloudy.

1806.	Six's Therm. without.			Thermometer without.			Thermometer within.			Barometer.*			Hygrometer.			Rain.
	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	
	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Inches.	Inches.	Inches.	Deg.	Deg.	Deg.	Inches.
January	55	28	42,5	55	29	42,7	58	48	53,1	30,13	28,65	29,52	96	78	88,3	1,837
February	56	30	43,4	56	32	43,8	57	46	52,4	30,28	29.33	29,84	96	72	87,6	0,535
March	56	26	42,7	56	26	43,0	58	47	53,6	30,45	28,90	29,72	95	74	87,0	1,328
April	64	31	45,7	64	33	46,1	59	50	54,9	30,47	29,38	30,02	94	71	84,0	0,245
May	75	43	57,8	73	46	58,8	67	56	61,9	30,43	29,40	29,89	97	65	80,7	1,020
June	83	46	62,5	83	52	63,5	71	62	65,9	30,53	29,60	30,10	94	53	75,9	0,505
July	81	52	64,5	81	53	64,0	69	63	65,8	30,21	29,51	29,81	100	66	81,3	4,889
August	80	51	64,5	79	54	64,9	70	62	67,3	30,20	29,25	29,85	91	68	79,0	2,087
September	73	44	59,5	73	44	59,6	68	59	63,0	30,27	29,53	30,02	98	71	82,7	1,920
October	65	35	53,2	64	38	53,7	62	53	59,1	30.33	28,66	29,87	97	70	84,9	0,793
November	59	36	49,1	56	37	49,7	59	53	56,5	30,30	29.05	29,76	97	78	88,8	2,551
December	57	39	48,0	56	40	48,8	59	52	55,8	30,28	28,72	29,58	97	79	88,3	2,717
Whole year			52,8			53,2			59,1			29,83			84,0	20,427

* The quicksilver in the bason of the barometer, is 81 feet above the level of low water spring tides at Somerset-house.

Variation of the Magnetic Needle.

1806.

June, 24° 8′, 6. West.